Discussion of Credit Risk Issues[1]

Nicholas M. Kiefer
Cornell University and OCC

February 5, 2009

[1]Prepared for the NISS/OCC Workshop: Exploring Statistical Issues in Financial Risk Modeling and Banking Regulation, DC February 5-6, 2009.

Introduction

This discussion will use a simple model to highlight some credit risk measurement issues.
Dennis Glennon: 2 issues, non iid data and large sample sizes
These are offsetting if handled correctly.

Samples

Let $d_{it} \in \{0, 1\}$ be the default indicator (asset i, period t), $D = \{d\}_{it}$ the full dataset, $r_t = \sum_i d_{it}$ the within period totals and R the collection of $\{r\}_t$. Suppose the marginal distribution of a default is bernoulli $p(d|\theta) = \theta^d(1-\theta)^{1-d}$.

Simplest model: Binomial

Maximum entropy (dist. least informative about joint realizations, data most informative about θ).

Defaults

The Basel II (B2) capital requirements are based on a one-factor model

$$v_{it} = \rho^{1/2} x_t + (1-\rho)^{1/2} \epsilon_{it}$$

where ϵ_{it} is the time and asset specific shock and x_t is the common time shock, inducing correlation ρ across asset values within a period. The random variables are standard normal and independent. Default occurs when the value drops below T. The marginal default rate is $\theta = \Phi(T)$. With $\rho = 0$ this leads to the binomial model.

Conditional Default Rates

The period t default probability is

$$\theta_t = \Phi[(T - \rho^{1/2} x_t)/(1-\rho)^{1/2}]$$

Within a period the defaults are independent conditioning on $\theta_t : p(d_{1t}, ...d_{nt}|\theta_t) = \prod_i p(d_{it}|\theta_t) \propto p(r_t|\theta_t)$, but conditioning on θ they are dependent: $p(d_{1t}, ...d_{nt}|\theta) = \int \prod_i p(d_{it}|\theta_t) p(\theta_t|\theta, \rho) d\theta_t$. The r_t remain independent across time. This distribution has lower entropy than the binomial.

Distribution $p(\theta_t|\theta,\rho)$

$$\begin{aligned}\Pr(\theta_t \leq A) &= \Pr(\Phi[(T-\rho^{1/2}x_t)/(1-\rho)^{1/2}] \leq A) \\ &= \Phi[((1-\rho)^{1/2}\Phi^{-1}[A] - \Phi^{-1}[\theta])/\rho^{1/2}]\end{aligned}$$

This could be generalized by adding autocorrelation in the systematic factor x_t, probably a good idea. That again reduces the entropy in the distribution; data are less informative about θ.

Some Implications

Theoretical implication: 2 views.
Doug Dwyer: The true model (marginally binomial with parameter θ known) will mostly overpredict defaults.
Nick Kiefer: Short time series of defaults will usually underpredict the long-run default rate θ.
Because the density $p(\theta_t|\theta,\rho)$ is right skewed for $\rho > 0$.

Practical Matters

In fact, the one-factor model with ρ close to values specified in Basel 2 implies too much temporal variation in default rates.
Specification tests based on whether rates are consistent with the model given estimates of θ will essentially never reject and are hence useless (the opposite side of Glennon's problem).
This might be mitigated by adding correlation in the systematic factor.

Additional Rambling

The validation exercise typically takes the prediction as a fixed set of numbers and does a statistical test on whether the realization is different from the prediction.
All randomness is due to the sampling distribution of the realized default rates.
A better approach: acknowledge uncertainty in the prediction as well as in the data.
Not usual since the models are estimated with such "precision."

Additional Rambling

Perhaps the precision is overstated due to some of the dependence issues, or due to unacknowledged model uncertainty or due to a model drift over time, leading to more recent data being more relevant than older data.

Represent uncertainty about θ by $p(\theta)$, based on all available information (historical data, expert opinion, etc.).

Use the model $p(r_{t+1}|\theta)$ to get the marginal distribution $p(r_{t+1}) = \int p(r_{t+1}|\theta)p(\theta)d\theta$. Predict using a loss function.

Lot's to do

Both in modeling and especially regarding validation.
Always interesting statistically - involves stepping away from the formalities of inference.
Involves judgment in ways that are hard to supress.