A Statistical View of Credit Risk

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Of what value a Rating?

- “Standard & Poor's said less than a year ago that Lehman continued to merit an A+ credit rating because of its ``exceptional liquidity, strong cost controls and **excellent risk management**.” Lehman went on to earn more than $150,000 per employee in 2006 and pay Chief Executive Officer Richard Fuld $40.5 million. ..... Investors dismissed Bear Stearns's corporate A+ rating when it sold $2.25 billion of five-year notes in August, after two of the firm's hedge funds collapsed because of bad bets on subprime mortgages. “

Wall Street Credit Costs Soar on Spread to U.S. Rates (Update2)

By Christine Harper Sept. 10, 2007 (Bloomberg) --

What I won't discuss

- Technicalities/details of contracts (e.g. Off balance sheet,…)
- Sophisticated exotic contracts/variations
- Details of how to price contracts
- Risk Neutral vs. Real world pricing.
- (Macro)economics

Concentrate Rather on

- Basic models people have suggested and used
- Pitfalls/advantages
- Can statistics can help (more?)

CREDIT
RISK

Standard Risk Measures

- Standard Deviation or Variance $E(\text{Loss}-\mu)^2$
- One-sided (semi) variance $E[(\text{Loss}-\mu)^+]^2$
- **Value at Risk:** Confidence level α=95% or 99%:

  VaR $= \min\{x;\ P[\text{Loss} \le x] \ge \alpha\}$
- Conditional Tail Expectation CTE $= E[\text{Loss} \mid \text{Loss} > \text{VaR}]$

(or Tail-VaR, Tail Conditional Expectation TCE, or Expected Shortfall)

Risk in a VaR-Regulated environment

- Suppose we apply **VaR constraint** e.g. $VaR_{0.99}=c$
- Returns are independent mean 0 (so total asset value is martingale) but distribution of returns flexible.
- Default when total return (or asset value) hits barrier at -b.
- Investors prefer firms with higher expected returns (conditional on non-default). Then with this objective function, CTE→∞

VaR Increases Risk: Boyle, Hardy& Vorst (2005), **Sharpening Sharpe Ratios (2002)**, Goetzmann, Ingersoll, Spiegel, Welch. Also Bernard and Tian(2008), Basak and Shapiro(2001).

Conjecture: If you constrain CTE and VaR, there is a risk measure and objective function such that risk→∞

Some Models for Credit Risk

Credit risk: *distribution* of loss due to failure of a financial agreement.

Credit derivative: security which allows the transfer of credit risk from one party to another

Credit Default Swap: in the event of *default*, the protection seller compensates protection buyer for loss. In return, buyer makes (quarterly) payments of the *swap spread*. (62 trillion in notional, Jan 2008)

(Like fire insurance on house but harder to diversify. A small number of periodically active arsonists go on occasional rampages and are responsible for many of the fires…and you don't need to own the house you insure*)

The **notional** amounts of over-the-counter derivatives continued to expand in the first half of **2008**, according to data from the Bank for International Settlements (BIS). They reached $684 trillion at the end of June, 15% higher than in December 2007. Interest-rate contracts, which account for the lion's share of the market, expanded by 17%. *Economist Nov 29, 2008*

Example: A default process

- The default process of a firm is a (random) stopping time τ with respect to some filtration* $\mathcal{F}_t$ so that the event $[\tau > t] \in \mathcal{F}_t$ for all t.
- Easy to generate random default events with given cumulative distribution function F using $\tau = F^{-1}(U)$ or

U[0,1] r.v.

$\tau = F^{-1}(\Phi(Y))$ where Y is N(0,1) and Φ is the standard normal c.d.f.

- When the default occurs, we need to model loss given default. (somewhat easier)

* The filtration depends on what information investors are assumed to have at time t. Guo, Jarrow & Zeng (2008), Duffie & Lando (2001) assume investors have incomplete and lagged information at discrete time points and show structural models can be viewed also as reduced-form.

Marginal Models ("industry Standard" just model joint distribution of τ)

- Generate $\tau_i = F_i^{-1}(\Phi(Y_i))$ where Y_i are (dependent) N(0,1) and Φ is the standard normal c.d.f.
- Model distribution of loss given default (or recovery rate) for each.

Issues:

- **Implementation:** easy to implement
  - Easy to introduce covariates, additional factors, etc.
  - Marginal distribution of default times F_i may be specified arbitrarily, or obtained from CDS spreads
- Loss given default independent?

Incorporating Dependence

- In all cases we can allow firms to "share" factors or incorporate covariates, latent variables.

e.g. $$Y_i = \rho_i M + \sqrt{1-\rho_i^2}\,\varepsilon_i$$

where ε_i are independent N(0,1) *idiosyncratic risk* factors and

M is a common (unobserved) N(0,1) market risk factor.

Issues:

- **Implementation:** easy.-Dimensionality=# common factors
  - Easy to introduce covariates, additional (latent) factors, etc.
- **Fit** to market data. Not with Gaussian copula(ρ>100%). Does not allow "contagion" (past defaults directly influence probability of future defaults)
- Parsimonious but **Rigid correlation structure: Gaussian copula induces too much independence especially in tails** (Embrechts et al.)

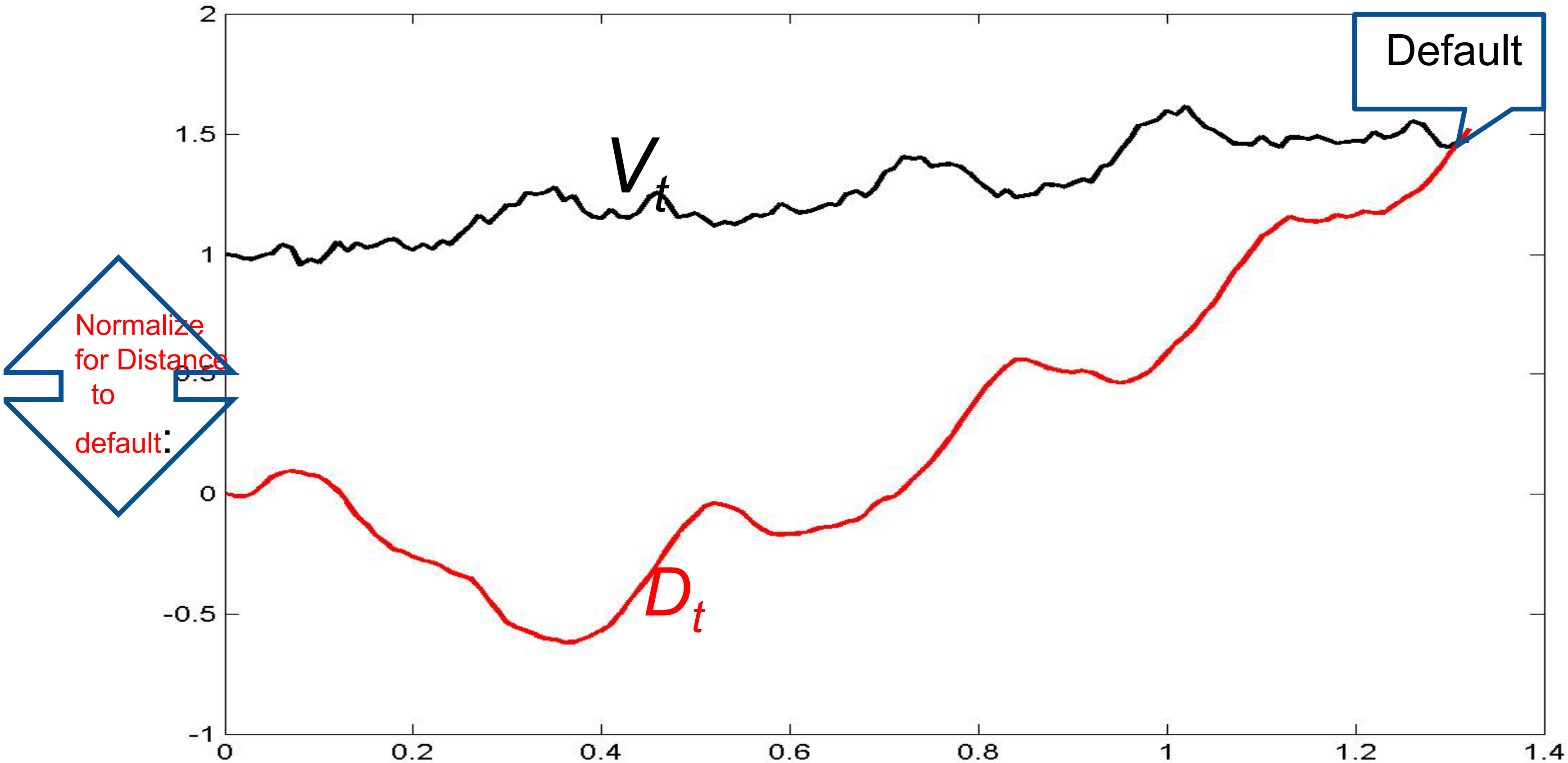
*Li (2000)

Dynamic Models:
1. Structural Models

- The gross market value firm's assets process V_t (which may or may not be observed by investors) is modelled, e.g. using a a (jump diffusion). (Merton's model: on log scale, V_t BM, variance σ^2)
- Default threshold D_t (liabilities) modelled as deterministic function or diffusion or integrated diffusion. The firm defaults at time $\tau=\inf\{t;\ V_t \leq D_t\}$
- V_t (and/or D_t) may be modelled with covariates, latent variables, missing information or discrete observations. Often D_t =constant.

Merton(1974), Black and Cox (1976), Zhou (1997), default of a company at first time when the firm-value falls below default boundary. Multivariate extensions -Hull et al. (2005) and Overbeck/Schmidt (2005)
-A Structural Model with Unobserved Default Boundary, (2006) T. Schmidt

Structural Model



Multivariate Case:

- $V_t^{(i)}$ modeled as diffusion (or (Geometric) Brownian motion)

$$dV_t^{(i)} = \mu^{(i)} \, dt + \sigma^{(i)} \, dW_t^{(i)}$$

- $D_t^{(i)}$ stochastic or deterministic function of t
- $W_t^{(i)}$ are correlated Brownian Motion
- Default of name (i) is first passage time of firm $V_t^{(i)}$ *to default barrier* $D_t^{(i)}$

Issues for Structural Models

- **Implementation:** only in special cases (1-2 dimensional case) are there closed form joint distributions for hitting times.
- **Calibration:** by Monte Carlo, difficult-(125-d) Challenging.
- **Fit to market data**. Defaults predictable (default rate near t=0 is zero*)
- Regime specific dependence on stock returns/volatility (Alexander & Kaeck)

* Duffie and Lando (2001)

An Alternative

Independent BM

- $dV_t^{(i)} = \mu^{(i)}(M_t)\, dt + g^{(i)}(\sigma_t)\, dW_t^{(i)}$ *
- Market factors M_t, σ_t satisfy mean-reverting (CIR-like) diffusion relationship e.g.

$$\mu^{(i)}(M_t) = \beta^{(i)} M_t, \quad g^{(i)}(\sigma_t) = \xi^{(i)} \sigma_t$$

$$dM_t = \kappa_M(\theta_M - M_t)\, dt + h(M_t)\, dW_t^{(M)}, \quad h(x) = 1 + a|x - \alpha| + b(x - \alpha)$$

σ_t^2 similar.

Could add Jumps to permit non-zero yield spreads (failure rate) at t=0

$$dV_t^{(i)} = \mu^{(i)}(M_t)\, dt + g^{(i)}(\sigma_t)\, dW_t^{(i)} + \delta \text{ (jump size at t)}$$

*Metzler & M

- **Equivalent to first passage time of** a time-changed Brownian motion, with time change

$$T(t) = \int_0^t \sigma_t^2 dt$$

to barrier at integrated market process,

$$D_t = d_0 + \int_0^t M_s ds$$

Advantage that we can simulate M_t , σ_t then calculate barrier crossing probabilities conditionally

Calibrates well across different tranches/maturities

Dynamic Models

2. Reduced-form Models

- Defaults are generated according to a non-homogeneous Poisson Process whose intensity process (the risk factor process) satisfies a jump diffusion:

P[default in $(t,t+\Delta t)$]= $\Lambda(X_t)\,\Delta t$

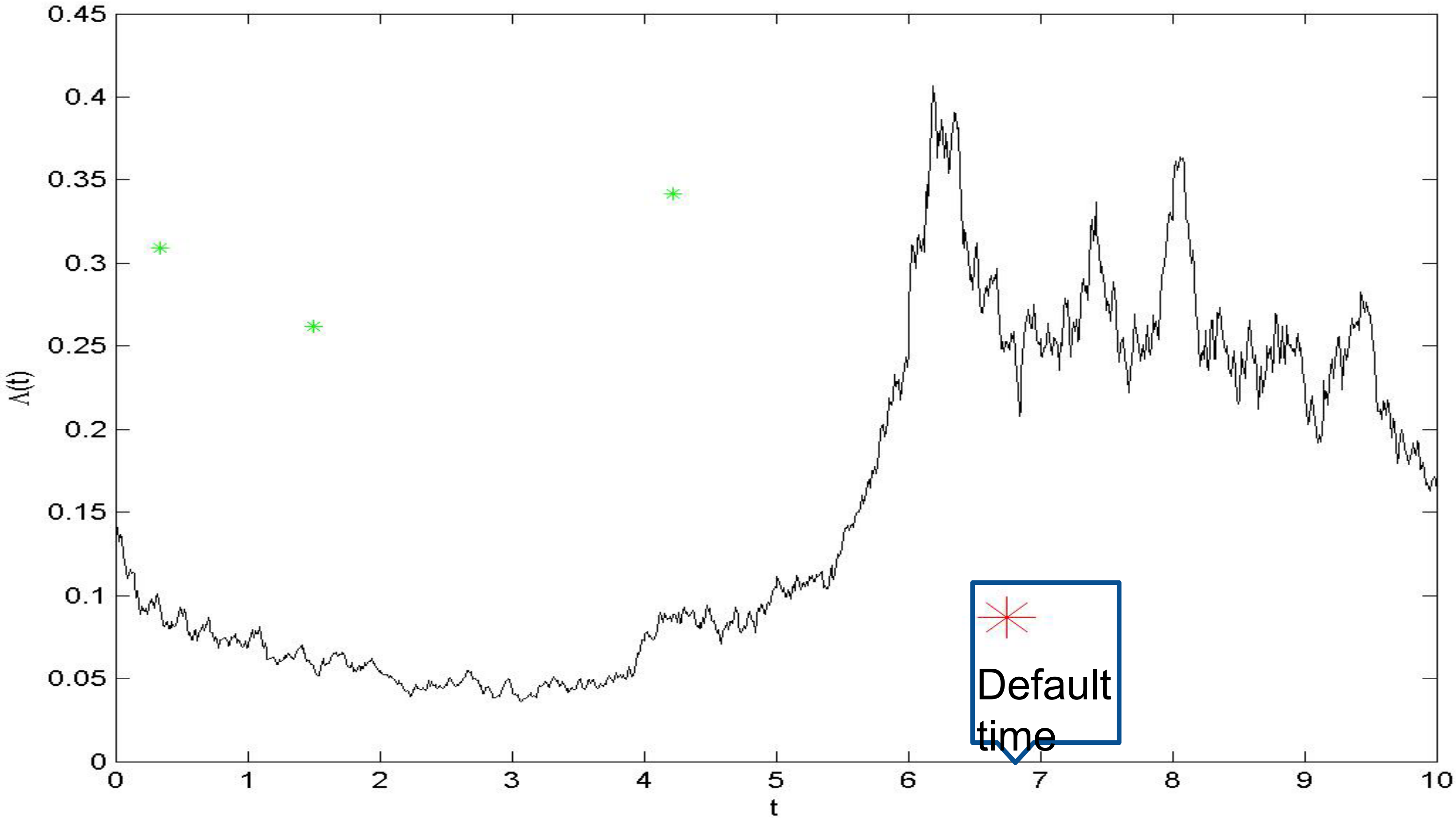
$dX_t = \mu(X_t)dt + \sigma(X_t)dW_t + \delta$ (jump size at t)

Jumps times: counting process intensity $\Lambda(X_t)$, sizes are i.i.d.

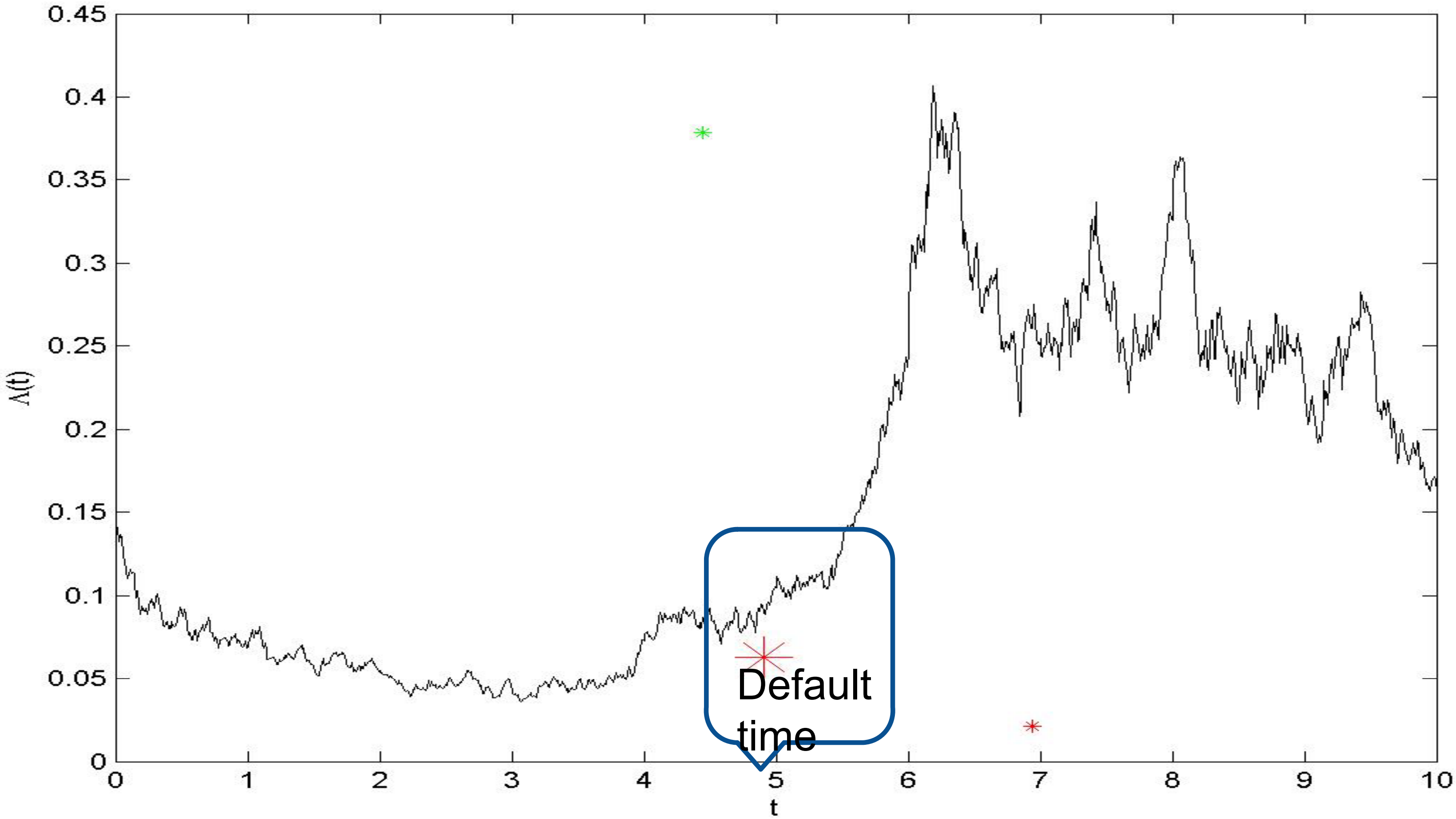
Affine*: $\mu(x) = K_0 + K_1 x$; $\sigma^2(x) = H_0 + H_1 x$; $\Lambda(x) = \Lambda_0 + \Lambda_1 x$

Jarrow and Turnbull (1992,1995) , Longstaff and Schwartz (1995), Duffie and Singleton (1999), Hull and White (2000) , Gieseke (2008), Litterman and Iben (1991), Madan and Unal , Lando , Duffie and Singleton, and Duffie., Jarrow, Lando, and Turnbull (1995),

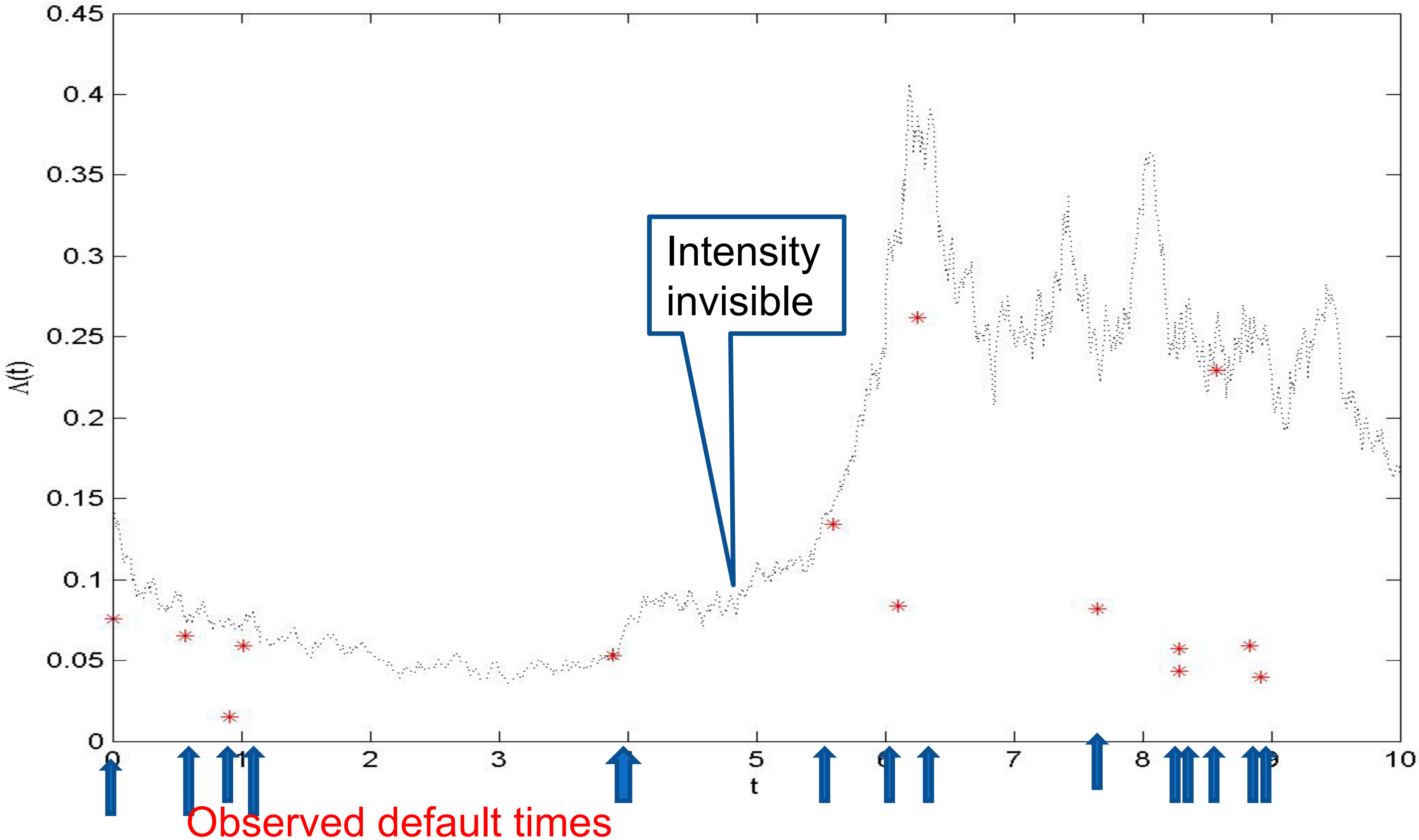
Reduced form models
Firm 1



Reduced form, Firm 2



Observation:



Dependence

- Firms can "share" intensity function or component thereof. Intensity can accommodate "*contagion**", (past defaults directly influences probability of future defaults), *covariates* (Individual firms-asset returns (KMV) or credit ratings (CreditMetrics)), *and risk factors* (Country/Sector factors) (frailty models**: share latent factors)
  - **Fit** to market data:
    - conditional independence given intensities
    - Parsimonious but more rigid correlation structure

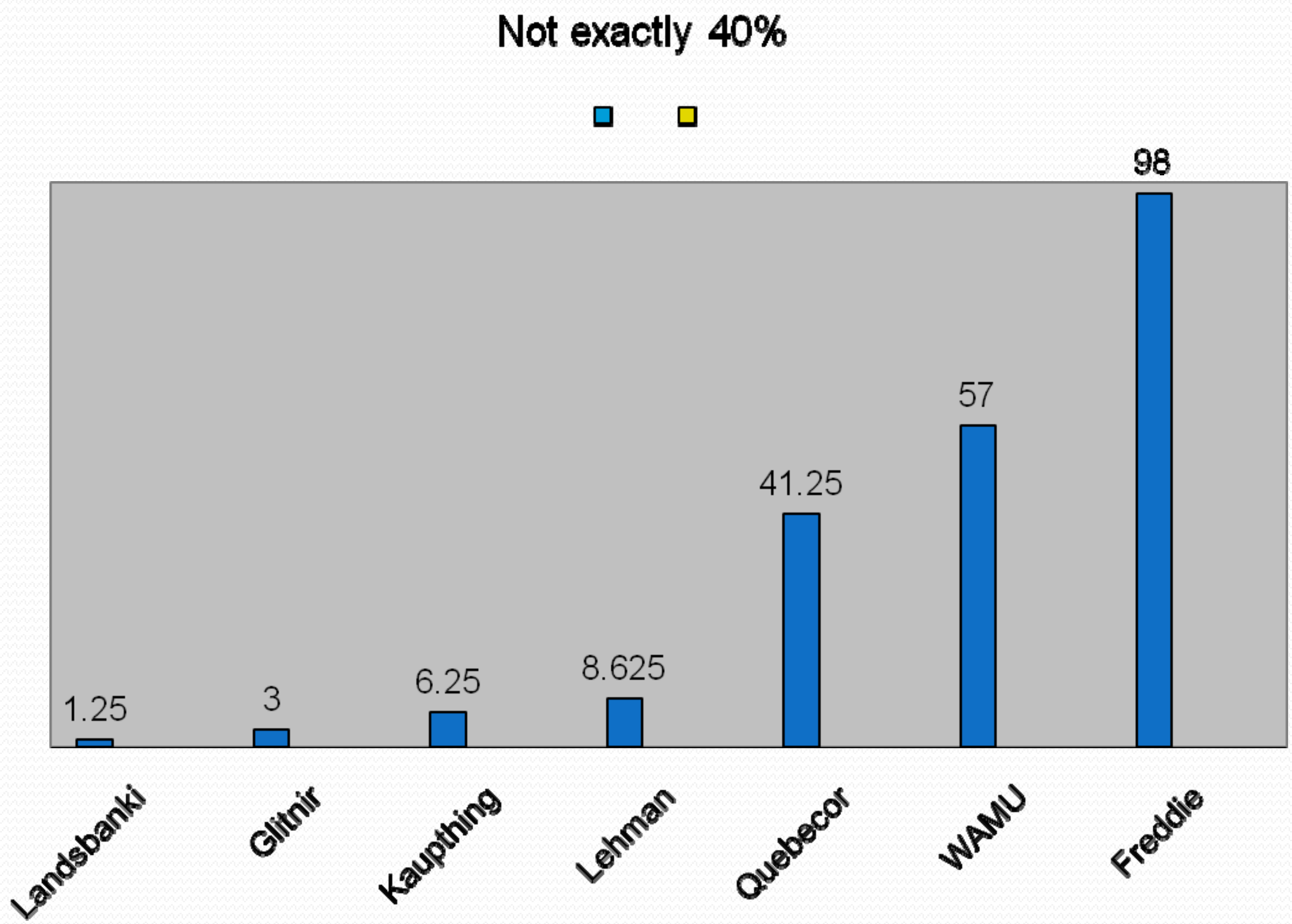
* Azizpour&Giesecke(2008) strong evidence of contagion

** Duffie, Eckner, Horel, Saita (2008)-latent variables are necessary

Default Correlation

- **Ambiguous Concept**
  - The correlation between the default times of two firms
  - The correlation between the distances to default (V_t-D_t assuming BM)
  - The correlation parameter in a Gaussian copula
  - The correlation between two firm default indicators over a fixed time horizon (Depends on time horizon, maximum possible value depends on correlation probabilities, may be <<1)
  - Correlation under extreme conditions changes anyway (panic and margin calls)
  - Risk not determined

Loss Given Default
2008 Recovery Rates



Speculative Remarks

- Too much attention paid to constructing models with **analytic tractability.** (Monte Carlo)
- **Google effect:** Speed of information flow- faster by orders of magnitude.
- **Model feedback & Fear factor:** (Finance is not Physics: endogeneity) psychology of defaults. Institutional aspects, leverage, margin calls
- **Traders as Gods.** Calibration vs. Estimation. Error estimates and sensitivities. Complexity of multivariate distributions. (1% change in ρ = 50% change in vol)
- Extremes of portfolios are driven by **tail dependence***. Modelling (tail) dependence the single most important feature in credit portfolio (psychological component)**

*Revisiting the edge, ten years on (2008) Chavez-Demoulin, Embrechts

** Correlation among house prices Boyle and ...

- How does the firm-value process of a firm near default differ from equity model? (Does an octogenarian invalid differ from a healthy 21 year-old)?
- **Linear Models:** Extraordinarily useful under "normal" conditions, they place too much faith in their own ability to forecast under extreme conditions.
- ***Where is the independence*?** (idiosyncratic factors). Proper use of statistics requires identifying i.i.d. residuals (hard to identify in finance)
- **Dynamically changing models:** parameters change to reflect higher volatility, greater correlation, feedback under extreme conditions, leverage effects.
- **Statistics , Moral-Hazard, and the Bernie Madoff effect**

"What is considered to most observers is not so much the annual returns-which though considered somewhat high for the strategy could be attributed to the firm's market making and trade execution capabilities-but the ability to provide such smooth returns with so little volatility" **Madoff Tops Charts; sceptics ask how, by M. Ocrant, MAR/Hedge 89, May 2001.**

A unique Model?

- For risk management, unique model unnecessary.
- Just as we use a battery of tests for random number generators, we can similarly subject risk strategies to a **portfolio of tests** motivated by competing models, driven by randomly generated scenario paths with random parameters.
- Test returns for statistical credibility.
- (move) structural models permit using Book values, debt, etc.
- Risk ≠ E(Loss) or a quantile. (Need to understand the distribution. Oversimplification → gaming)

Scenario generators should accommodate

- Feedback effects*
- Model uncertainty-change under stress. Models that cannot be rejected by goodness-of-fit retained.
- Parameter uncertainty
- Potential biases due to moral-hazard

28.07.2008 18:03 **IMF says credit crisis spreading in 'negative feedback loop' UPDATE**
(Updates with further quotes from IMF's Global Financial Stability report)
WASHINGTON (Thomson Financial) - The credit crisis stemming from the U.S. housing slump has triggered a 'negative feedback loop' in the global economy that poses risks for growth, the International Monetary Fund said Monday.
'Global financial markets continue to be fragile and indicators of systemic risks remain elevated,' the IMF said in an update of its April report on global financial stability.
A year after the financial markets were battered by the U.S. subprime, or high-risk, home mortgage crisis, those expected losses have now been 'largely acknowledged' by financial institutions, but the risk contagion has spread to other forms of credit.
'Credit quality across many loan classes has begun to deteriorate, with declining house prices and slowing economic growth,' the 185-nation institution said

Ugly feedback loop: Debt fears, stock fears, recession fears 11:29 AM, October 24, 2008
...The slump has been driven by hedge funds dumping assets amid record losses and investor withdrawals, and concern that governments from Argentina to Pakistan may default in a global recession.
This is the ugly feedback loop: As credit default swap prices rise, expectations of financial calamity surge, which drives stocks lower. And the continuing crash in stock prices threatens to turn a recession into something much worse by destroying consumers' wealth and their purchasing power. A severe recession, in turn, would assure more debt defaults.
The prices of swaps on **General Motors** Corp. debt imply a 93% chance of default in the next five years, Bloomberg said, based on JPMorgan Chase & Co. model.