Less restrictive regularity conditions for the generalised Jackknife variance estimator for unequal probability sampling

López-Escobar, Emilio

University of Southampton, United Kingdom.

ITSEW 2009. Tällberg, Sweden, 2009

Outline

1. Introduction to the Campbell-Berger-Skinner Jackknife Variance Estimator
   - Basic Features
   - Some Notation
   - Variance Estimator

2. Asymptotic Design-Consistency
   - About the One Stage Sampling Regularity Conditions
   - Adjustments for More than One Stage Sampling

3. Summary

Campbell-Berger-Skinner Jackknife Variance Estimator.

- Based on analogy between jackknife and linearization (Campbell, 1980).
- Consistent for a broad class of point estimators (Berger-Skinner, 2005).
- Allows:
  - Any without-replacement unequal probability sampling.
  - Stratification.
- Showed improved performance compared with existing jackknife estimators (naturally includes FPC, absent in standard methods).

Campbell-Berger-Skinner Jackknife Variance Estimator.

- Design-Based approach.
- Finite population $\mathcal{U}$ (size N), sample $\mathcal{S}$ (size n), $q = 1, .., Q$ variables.
- π_k and π_{kl}, 1st and 2nd order inclusion probabilities.
- Parameter:
  - $\theta = g(\mu_1, \dots, \mu_Q)$, $g(\cdot)$ smooth differentiable.
  - μ_q mean of values y_{q_k}.
- Point Estimator:
  - $\widehat{\theta} = g(\widehat{\mu}_1, \dots, \widehat{\mu}_Q)$.
  - $\widehat{\mu}_q = \sum_{k \in \mathcal{S}} \frac{y_{q_k}}{\hat{N} \pi_k}$ estimator of μ_q (Hájek, 1971).

Campbell-Berger-Skinner Jackknife Variance Estimator.

The Variance Estimator:

$$\widehat{Var}(\hat{\theta}) = \sum_{k \in \mathcal{S}} \sum_{l \in \mathcal{S}} \frac{\pi_{kl} - \pi_k \pi_l}{\pi_{kl}} \varepsilon_{(k)} \varepsilon_{(l)}$$

with:

$$\varepsilon_{(k)} = (1 - w_k)(\widehat{\theta} - \widehat{\theta}_{(k)})$$

$$w_k = \frac{1}{\hat{N}\pi_k} = \left\{ \left(\sum_{k \in \mathcal{S}} \frac{1}{\pi_k} \right) \pi_k \right\}^{-1} \text{ (Hájek, 1971)}$$

$\widehat{\theta}_{(k)} = g(\widehat{\mu}_{1(k)}, \ldots, \widehat{\mu}_{Q(k)})$, the same as $\widehat{\theta}$ but excluding k-th observation.

- Under some **regularity conditions**, consistent for **one-stage sampling**.

Asymptotic Design-Consistency Regularity Conditions.

- Established regularity conditions (8):
  - Consistency of the linearization variance estimator.
  - None of the weights w_k can approach 1.
  - Smooth function $g(\cdot)$ - Lipschitz (Hölder) continuity of order $\delta > 0$.
  - The following restriction on $D_{kl} = \frac{\pi_{kl} - \pi_k \pi_l}{\pi_{kl}}$,

$$\sum_{k \in \mathcal{S}} \sum_{\substack{l \in \mathcal{S} \\ l \neq k}} \left(D_{kl}^{+}\right)^2 = O_p(1), \quad \textbf{(1)}$$

$$\text{where } D_{kl}^{+} = \begin{cases} D_{kl}, & \text{if } D_{kl} > 0, \\ 0, & \text{otherwise.} \end{cases}$$

(1) Too restrictive for **more than 1 stage**

$\Longrightarrow$ **condition holds when n is bounded**.

(e.g. Large fixed N, Hartley-Rao, 1962)

Less Restrictive Regularity Conditions.

Adjustment to (1)

$$n^{-\beta} \sum_{k \in \mathcal{S}} \sum_{\substack{l \in \mathcal{S} \\ l \neq k}} \left(D_{kl}^{+}\right)^{2} = O_p(1),$$

with

$$0 \leq \beta < 2\delta,$$

and $0 < \delta \leq 1$ the Lipschitz (Hölder) condition order.

- **This condition allows** $n \longrightarrow \infty$ for 2 stage sampling.

Less Restrictive Regularity Conditions.

- Smoothness slightly modified (Lipschitz condition of order $0 < \delta \leq 1$).

$\Downarrow \quad \Downarrow \quad \Downarrow \quad \Downarrow \quad \Downarrow \quad \Downarrow$

- Less restrictive and still design-consistent.
- Extends to more than one stage.

Example: New regularity conditions hold for *1 Stage Cluster SRS*, and for the *2 stage SRS-SRS* sampling designs ($\beta = 1, \delta > \frac{1}{2}$).

Further work: Self-weighted 2 stage sampling, and comparison with standard jackknifes.

Summary

- Regularity conditions on π_{kl} (Berger-Skinner, 2005)
  - Too restrictive, because n need to be bounded for 2 stage sampling.
- We propose **new less restrictive** conditions:
  - Hold for 2 stages when $n \longrightarrow \infty$
    $N \longrightarrow \infty$