A Cost-Efficient Approach to Wireless Sensor Network Design

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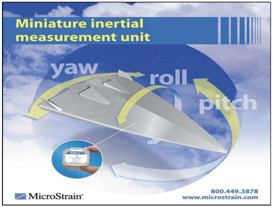
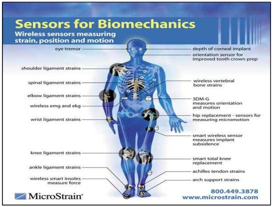
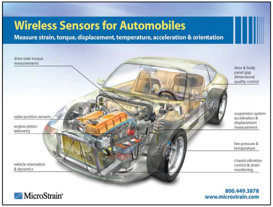
Wireless Sensor Networks: Applications

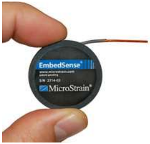


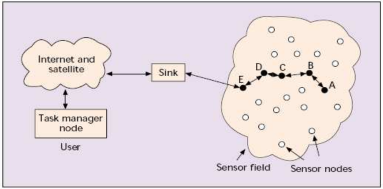




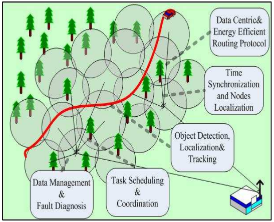
Wireless Sensor Networks: Structure

- Large number of small, inexpensive, low-power **sensors**:
  - deployed in a regular pattern (e.g. grid), at random ("smart dust"), or ad hoc
  - collect information about the surrounding environment
  - have some data processing capabilities
  - communicate with other sensors and remote (fusion) center
- Fusion center processes data from sensors and makes a global (and more precise) situational assessment

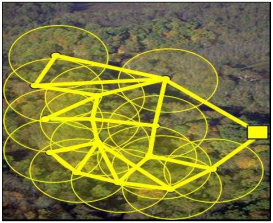




Wireless Sensor Networks: Design



Coverage



Connectivity

Constraints

- Low power: minimize data processing and communications costs
- Communication to center expensive ⇒ distributed algorithms

Wireless Sensor Networks: Cost

Trade-offs:

- network cost and reliability
- sensor cost and quantity

A Cost-Efficient Wireless Sensor Network Design

Objective

Minimization of the overall **network cost** (determined by the sensor's sensing and communication capabilities), while maintaining:

- **coverage** constraint - most of the region of interest is within the sensing range of at least one sensor,
- **connectivity** constraint - each sensor can communicate with any other sensor either directly or through its neighbors with high probability.

Framework

- enforces both constraints simultaneously;
- allows for unreliable sensors;
- can be extended to heterogeneous networks;
- can incorporate any performance measure as a cost function.

Related Work

Problem of the region coverage is comprehensively studied by P.Hall(1988).
General connectivity results are covered by P.Gupta and P.Kumar(1998), C. Bettstetter(2002), M. Penrose(2003), and references therein.

Sensor Networks Design

- coverage via the minimal covering (S. Shakkottai, 2003), **but** that precludes random deployment of sensors;
- minimize energy cost under a lifetime constraint (V. Mhatre, 2005);
- minimize target localization error bound (W. Wang, 2006);
- minimize network search cost (J. Ahn, 2007).

However, these cost functions do not incorporate the sensor characteristics. themselves, such as the sensing and communications capabilities.

Problem Formulation

Q.: How does one minimize the cost of the network under constraints while taking cost of sensing and communications into accounts?

Design approach:

- deploy n sensors at random over region $X \in \mathbb{R}^2$ (WLOG $X = [0, 1] \times [0, 1]$)
- sensing range s, communications range r are taken from a set of feasible values $D \subset (0, \infty) \times (0, \infty)$. Here the sensor design space D is given by the Cartesian product $[S_L, S_U] \times [R_L, R_U]$;
- the cost function for each sensor $C(s, r)$ is a *positive, continuous, non-decreasing function* in both arguments; i.e.

$$C(s, r) \geq 0, \quad \partial C(s, r)/\partial s \geq 0, \quad \partial C(s, r)/\partial r \geq 0;$$

- each sensor performs its task with probability $p \in (0, 1)$ and fails to do so with probability $1 - p$.

Problem Formulation

Objective: to determine the number n and the type (in terms of s and r) of sensors to be *randomly* deployed over X to minimize the total network cost,

$$\min_{n,(s,r)\in D} nC(s,r) \tag{1}$$

subject to:

- (a)**Coverage constraint**: the expected area of the region covered (out of the total of 1) exceeds a pre-specified threshold $1-\epsilon,\ \epsilon > 0$;
- (b)**Connectivity constraint**: the probability that there is a communication path between any two sensors exceeds another pre-specified threshold $1-\delta,\ \delta > 0$.

Define $n(s,r) = \min\{n : \text{constraints (a) and (b) are satisfied}\}$, then the optimization problem reduces to:

$$\min_{(s,r)\in D} n(s,r)C(s,r)\ . \tag{2}$$

General Solution for Convex Objective Function

If the objective function is convex in both s and r, a global minimum exists and can be determined by solving for the point $(\tilde{s}, \tilde{r}) \in D$ satisfying

$$\frac{\partial(n(s,r)\,C(s,r))}{\partial s}\bigg|_{(\tilde{s},\tilde{r})} = 0, \quad \frac{\partial(n(s,r)\,C(s,r))}{\partial r}\bigg|_{(\tilde{s},\tilde{r})} = 0, \tag{3}$$

and the Hessian matrix of second partial derivatives being positively definite,

$$\frac{\partial^2 C(s,r)}{\partial s^2}\bigg|_{(\tilde{s},\tilde{r})} > 0, \qquad \frac{\partial^2 C(s,r)}{\partial r^2}\bigg|_{(\tilde{s},\tilde{r})} > 0$$

$$\left(\frac{\partial^2 C(s,r)}{\partial s^2}\frac{\partial^2 C(s,r)}{\partial r^2} - \frac{\partial^2 C(s,r)}{\partial r \partial s}\frac{\partial^2 C(s,r)}{\partial s \partial r}\right)\bigg|_{(\tilde{s},\tilde{r})} > 0. \tag{4}$$

If such a point $(\tilde{s}, \tilde{r})$ is not feasible, then the solution is given by a point on the boundary of the design space D. Ultimately, the solution is determined by the form of functions $n(s,r)$ and $C(s,r)$.

The Coverage Constraint

Focusing only on the coverage constraint, the problem becomes

$$\min_{n,s\in S} nC(s), \quad \text{s.t.} \quad \mathbb{E}(\text{area not covered}) \leq \epsilon,$$

where $C(s)$ is a positive, continuous, non-decreasing function of s. From the theory of coverage processes (Hall,1988):

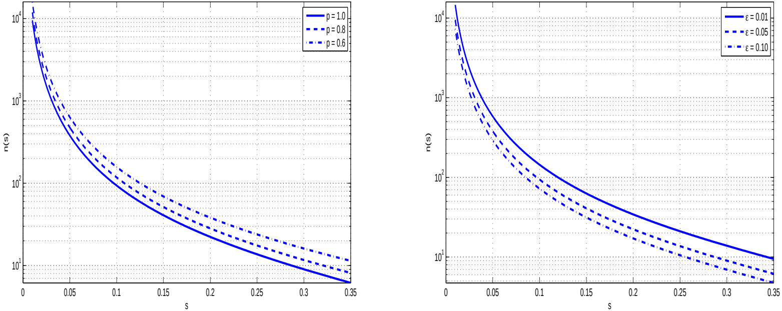
$$\mathbb{E}(|\tilde{X}|) = \left(1 - \frac{p\pi s^2}{|X|}\right)^n . \tag{5}$$

$\text{Var}(|\tilde{X}|) \to 0$ as a function of n sufficiently fast, we can assume that the *actual* area not covered by the network deviates little from the above expression. The smallest n that satisfies the constraint $\mathbb{E}(|\tilde{X}|) \leq \epsilon$ is

$$n(s) = \frac{\log(\epsilon)}{\log(1 - p\pi s^2)} . \tag{6}$$

Number of sensors as a function of sensing radius

Figure 1: Number of sensors $n(s)$ as a function of sensing radius s for different values of the active state probability p with $\epsilon = 0.05$ (left panel); and for different values of ϵ with $p = 1$ (right panel).



Interior Point Solution

The conditions for an **interior point minimum** to exist at point $\tilde{s} \in (S_L, S_U)$:

$$\begin{cases} n'(\tilde{s})C(\tilde{s}) + n(\tilde{s})C'(\tilde{s}) = 0 \\ n''(\tilde{s})C(\tilde{s}) + 2n'(\tilde{s})C'(\tilde{s}) + C''(\tilde{s})n(\tilde{s}) > 0, \end{cases} \quad (7)$$

where

$$n'(s) = 2\frac{\log(\epsilon)\pi sp}{\left(\log\left(1-\pi s^2 p_s\right)\right)^2\left(1-\pi s^2 p\right)} \quad (8)$$

$$n''(s) = 2\frac{\log(\epsilon)\pi sp\,\left[4\pi ps^2 + (1+\pi s^2 p)\log\left(1-\pi s^2 p\right)\right]}{\left(\log\left(1-\pi s^2 p\right)\right)^3\left(1-\pi s^2 p\right)^2} \quad (9)$$

If such a point is not feasible ($\tilde{s} \notin S$) then the solution would occur either at S_L or at S_U.

A Cost Function Example for Coverage

An example of an **additive cost** function:

$$C(s) = c_0(1 + c_1 s^{\beta_1}), \quad c_0, \; c_1, \; \beta_1 > 0 \;, \tag{10}$$

where c_0 represents the *fixed* cost of the sensor independent of its sensing capabilities (WLOG $c_0 = 1$), while the second term is the *variable* cost which is an increasing function of the sensing radius.
The network-wide cost and its first derivative are:

$$\begin{aligned} n(s)\, C(s) &= \frac{\log(\epsilon)}{\log(1 - \pi\, s^2\, p)} \left(1 + c_1\, s^{\beta_1}\right), \\ ((n(s)\, C(s)))' &= \frac{\log(\epsilon)}{s\; \log(1 - \pi\, s^2\, p)} \left(-b(s) + c_1(\beta_1 - b(s)) s^{\beta_1}\right) \;, \end{aligned}$$

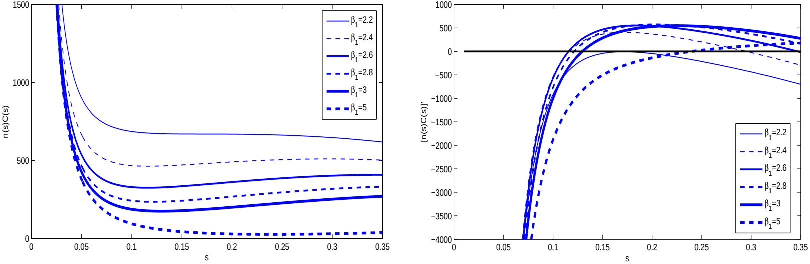
where

$$b(s) = \frac{-2s^2 \pi p}{(1 - \pi s^2 p) \log(1 - \pi s^2\, p)} \;. \tag{11}$$

Optimal Solution for different values of β_1

The existence of an interior solution $\tilde{s} \in (S_L, S_U)$ depends on c_1 and β_1.

- For $0 \leq \beta_1 < 2$, the overall network cost $n(s)C(s)$ is a decreasing function of s, so the optimal design is achieved for $s = S_U$.
- For $\beta_1 \geq 2$, the convexity and interior vs. boundary solution depend on the choice of c_1.



Netw cost function $n(s)C(s)$ (left panel) and its first derivative (right panel) with coefficients $c_0 = 1$ and $c_1 = 1000$ for different values of β_1, with $p = 1$, $\epsilon = 0.05$, $S = [0.01, 0.035]$.

The Connectivity Constraint

When the connectivity constraint is considered alone, the problem becomes:

$$\min_{n,r \in R} nC(r), \quad \text{s.t.} \quad \mathbb{P}(\text{network is not connected}) \leq \delta,$$

where $C(r)$ is a positive, continuous, non-decreasing function of r.

Let $G(n, r)$ denote the graph with node set corresponding to the sensors and edges between all pairs of nodes whose distance is less than or equal to r. Then the network is considered connected if the graph G is connected.

Results for the minimum sufficient number of sensors

- Kumar-Gupta Connectivity Bound (1998)

$$\pi r^2 n_{GK}(r) - \log(n_{GK}(r)) = -\log(\delta/4)\ . \qquad (12)$$

- Bettstetter Connectivity Bound (2002)

$$n_B(r)\log\left(1 - \exp(-n_B(r)\,\pi\, r^2)\right) = \log\delta\ . \qquad (13)$$

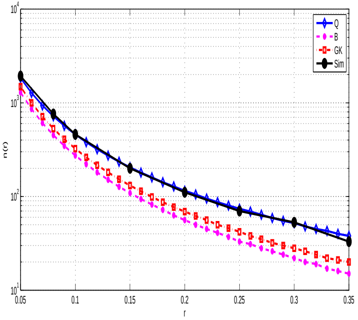
- **New Connectivity Bound**

$$n(r) = \frac{q(\delta)}{\pi\, r^2\, p}\ , \text{where} q(\delta) = \gamma_0\delta^{-1} + \gamma_1 + \gamma_2\delta^1, \qquad (14)$$

where the coefficients $\gamma_0 = 0.0397, \gamma_1 = 14.1208, and \gamma_2 = -9.4352$ were obtained from the least squares fit to simulation data ($R^2 = .975$).

Number of sensors as function of communication radius

Figure 3: Number of sensors n as function of communication radius r for $\delta = 0.05$ (left panel) and $\delta = 0.1$ (right panel).



A Cost Function Example for Connectivity

An **additive sensor cost** function of the form

$$C(r) = c_0(1 + c_2\, r^{\beta_2}), \tag{15}$$

where the variable component depends only on the communication radius r.

WLOG assume $c_0 \equiv 1$, and $c_2 \geq 0$, $\beta_2 \geq 0$. The corresponding network-wide cost function and its first derivative are:

$$\begin{aligned} n(r)\, C(r) &= r^{-2} + c_2\, r^{\beta_2 - 2}, \\ (n(r)\, C(r))' &= r^{-3}\left(-2 + c_2(\beta_2 - 2) r^{\beta_2}\right). \end{aligned}$$

Optimal Solution for different values of β_2

- For $0 < \beta_2 \leq 2$, the network cost is a decreasing function of r and the solution occurs at $r = R_U$.
- For $\beta_2 > 2$, there are values of c_2 for which an interior extremum point $\tilde{r}$ of equation $(n(r)\,C(r))' = 0$ exists:

$$\tilde{r} = (0.5\,(\beta_2 - 2)c_2)^{-1/\beta_2} \quad . \tag{16}$$

- For $\beta_2 > 3$, the function is convex, the extremum point is unique, and (16) provides the solution to the optimization problem, as long as c_2 is such that $\tilde{r} \in R$.

$$r_o = \begin{cases} R_L, & \tilde{r} \leq R_L \\ R_U, & \tilde{r} \geq R_U \\ \tilde{r}, & \tilde{r} \in (R_L, R_U) \end{cases} \tag{17}$$

- For $2 < \beta_2 \leq 3$, the function is not convex, the minimum is achieved either at the extremum point or at the boundary.

Optimal Solution for different values of β_2 and c_2

Table 1: The optimal communication radius r (rounded to 3 significant digits) and the number of sensors n for different values of c_2 and β_2 with $p = 1$, $\delta = 0.05$, $R = [0.01, 0.35]$.

β_2	2.2		2.4		2.8		3		5	
c_2	r	n	r	n	r	n	r	n	r	n
100	.350	39	.290	58	.268	66	.272	64	.350	39
500	.169	165	.147	219	.151	208	.159	187	.266	67
1e3	.124	310	.110	390	.118	340	.126	297	.232	88
5e3	.059	1337	.056	1488	.067	1073	.074	867	.168	167
1e4	.044	2511	.042	2651	.052	1760	.059	1376	.146	221

Joint Coverage and Connectivity Optimization

When solving joint coverage and connectivity optimization problem, $n(s,r)$ is the smallest n that satisfies the connectivity$(n(r) \equiv n_2(r))$ and coverage $(n(s) \equiv n_1(s))$ constraints for given r and s and depends on D, ϵ, δ, and p.

Case 1: the coverage constraint dominates, $n_1(s) > n_2(r)$ for all $(s,\ r) \in D$. The optimal value of r is clearly $\tilde{r} = R_L$, so the optimal sensing radius $\tilde{s}$ minimizes $n_1(s)\, C(s, R_L)$.

Case 2: the connectivity constraint dominates, $n_1(s) < n_2(r)$ for all $(s,\ r) \in D$, so the optimal sensing radius $\tilde{s} = S_L$ and communication radius $\tilde{r}$ minimizes $n_2(r)\, C(S_L, r)$.

Joint Coverage and Connectivity Optimization

Case 3: $D = D_1 \cup D_2$, with $n_1(s) \geq n_2(r)$ for all $(s,\ r) \in D_1$, and $n_1(s) \leq n_2(r)$ for all $(s,\ r) \in D_2$, $\rightarrow$ the optimal values s and r will lie either on the boundary or on the curve $n_1(s) = n_2(r)$.

Solving $n_1(s) = n_2(r)$ for r gives

$$r(s) = \left(\frac{q(\delta)}{\pi\, p \log(\epsilon)} \log(1 - \pi\, s^2\, p) \right)^{1/2}. \tag{18}$$

The optimization problem then reduces to

$$n_1(s)\, C(s, r(s)), \text{over the set } (s, r(s)) \in D.$$

A Joint Cost Function Example

Consider a sensor cost function of the form

$$C(s, r) = c_0(1 + c_1 s^{\beta_1} + c_2 r^{\beta_2}), \tag{19}$$

with $c_0 = 1$ without loss of generality and non-negative parameters c_1, c_2, β_1, β_2.

Case 1: $n_1(S_U) > n_2(R_L)$, and the extremum points of the function $n_1(s)\, C(s, R_L)$ are solutions to the equation:

$$b(s)(1 + c_1 s^{\beta_1} + c_2 R_L^{\beta_2}) - c_1 \beta_1 s^{\beta_1} = 0 \ , \tag{20}$$

where $b(s)$ is defined in (11).

Case 2: $n_2(R_U) > n_1(S_L)$, and the extremum point of $n_2(r) C(S_L, r)$ is defined by

$$r^{\beta_2} = \frac{2(1 + c_1\, S_L^{\beta_1})}{c_2(\beta_2 - 2)} \tag{21}$$

Note, extremum points only exist if $\beta_2 > 2$.

Case 3: Plugging in the expression for $s(r)$ into $n_2(r)\, C(r, s(r))$ and taking the derivative with respect to r gives an extremum point equation

$$-2 + c_2(\beta_2 - 2)r^{\beta_2} + s^{\beta_1}(r)c_1 \left(-2 + r\beta_1 \frac{s'(r)}{s(r)} \right) = 0 \ , \qquad (22)$$

where

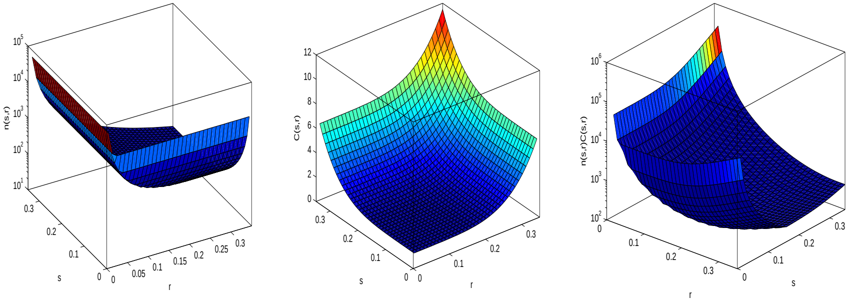
$$s'(r) = -\frac{r \ \log(\epsilon)}{s(r)\ q(\delta)} \exp\left(\pi\, r^2\, p\, \log(\epsilon)/q(\delta)\right) .$$

Optimal Solution for different values of β_1 and β_2

Table 2: Optimum values of the sensing radius s and communication radius r, and the corresponding number of sensors n, for different values of β_1 and β_2 with $c_1 = c_2 = 10^3$, $\epsilon = 0.05$, $\delta = 0.05$, and $p = 1$.

β_1	2.0			2.4			3.0		
β_2	s	r	n	s	r	n	s	r	n
2.0	.155	.347	39	.118	.262	67	.129	.287	56
2.4	.050	.110	380	.046	.101	451	.050	.110	380
2.8	.055	.121	315	.050	.110	380	.052	.114	354
3.0	.059	.130	273	.055	.121	315	.055	.121	315
5.0	.110	.244	78	.090	.199	117	.095	.210	105

Figure 4: Number of sensors $n(s, r)$ (left panel), sensor cost function $C(s, r)$ (middle panel), and overall network cost $n(s, r)C(s, r)$ (right panel) with $c_1 = c_2$ =1e3, $\beta_1 = \beta_2 = 5$, $\epsilon = \delta = 0.05$, and $p = 1$. The optimal design is $s = 0.104$, $r = 0.230$, $n = 87$.



Extensions and Discussion

The proposed approach should be regarded as part of any feasibility study during the planning stages for the deployment of a wireless sensor network, when decisions about its capabilities and cost are considered.

Our framework can accommodate:

- separate costs and coverage requirements for cluster heads of hierarchical networks;
- the reliability parameter linked to the sensor characteristics s and r and its cost;
- the probability of a sensor actively sensing (p_1) is not the same as the probability of being able to communicate (p_2);
- accuracy of target localization or field estimation.

An additional technical contribution is the derivation of a new simple bound on the probability of a network being connected,

Useful References

1. P. Hall, Introduction to the theory of coverage processes, 1988.
2. M. Penrose, Random Geometric Graphs, 2003.
3. P. Gupta and P. Kumar, "Critical power for asymptotic connectivity in wireless networks," 1998.
4. S. Shakkottai, R. Srikant, and N. Shroff, "Unreliable sensor grids: Coverage, connectivity, and diameter," 2003.
5. N. Katenka, E. Levina, and G. Michailidis, "A Cost-Efficient Approach to Wireless Sensor Network Design," (submitted), 2008.
6. C. Bettstetter, "On the minimum node degree and connectivity of a wireless multihop network," 2002.

THANK YOU VERY MUCH

Questions/Comments/Suggestions