Bayesian Reliability Analysis: Statistical Challenges from Science-Based Stockpile Stewardship

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May 22, 2008

LA-UR-08-3570

Acknowledgments

- Christine Anderson-Cook
- Mike Hamada
- Aparna Huzurbazar
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Outline

Science-Based Stockpile Stewardship

The goal of science-based stockpile stewardship is the assessment of safety and reliability in aging warheads in the absence of nuclear testing.

Surveillance Transformation

Develop and advance the analytical capabilities required to perform computational predictions of stockpile performance, reliability, end-of-life, safety, survivability, use control; and to provide risk-based responsiveness for future replacement and refurbishment decisions.

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- Uncertainty quantification

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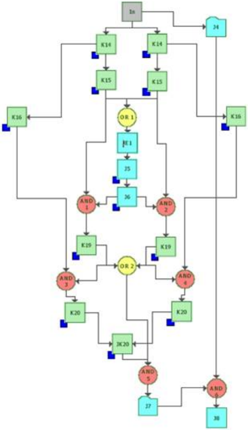
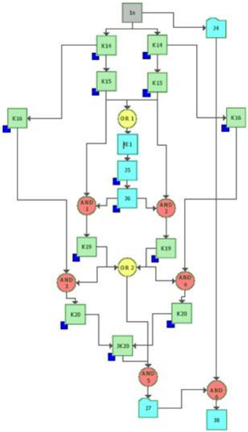
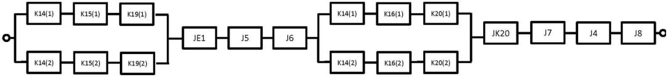
- Uncertainty quantification
- Planning

B61 Case Study: Block Diagrams



B61 Case Study: Block Diagrams





System Reliability Equation

$$
\begin{aligned}
R_{SYS} = {} & R_{JK20} R_{J7} R_{J4} R_{J8} R_{JE1} R_{J5} R_{J6} (\\
& R_{K14(1)} R_{K14(2)} R_{K16(1)} R_{K16(2)} R_{K20(1)} R_{K20(2)} R_{K15(1)} R_{K15(2)} R_{K19(1)} R_{K19(2)} \\
& - R_{K14(1)} R_{K14(2)} R_{K16(1)} R_{K16(2)} R_{K20(1)} R_{K20(2)} R_{K15(2)} R_{K19(2)} \\
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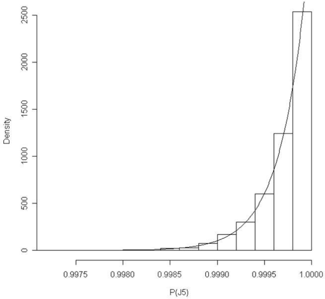
& - R_{K14(1)} R_{K14(2)} R_{K16(2)} R_{K20(2)} R_{K15(1)} R_{K15(2)} R_{K19(1)} R_{K19(2)} \\
& + R_{K14(1)} R_{K14(2)} R_{K16(1)} R_{K20(1)} R_{K15(2)} R_{K19(2)} \\
& + R_{K14(1)} R_{K14(2)} R_{K16(2)} R_{K20(2)} R_{K15(1)} R_{K19(1)} \\
& + R_{K14(1)} R_{K16(1)} R_{K20(1)} R_{K15(1)} R_{K19(1)} \\
& + R_{K14(2)} R_{K16(2)} R_{K20(2)} R_{K15(2)} R_{K19(2)})
\end{aligned}
$$

Overview of Methodology

- Estimate reliability for each component
  - Point estimate
  - Measure of uncertainty (credible interval)
- Combine component reliability distributions using Monte Carlo to get system reliability distribution

Binary Data

No failures in 3513 tests



Multilevel Data

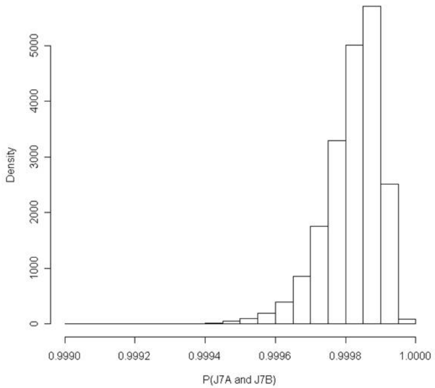
Data on components and collections of components

- Data J7A: 383 variables measurements. Requirement$(-4.5, 4.5)$
- Data J7B: 468 variables measurements. Requirement$(-3.8, 3.8)$
- There are 383 tests with both measurements, and 85 with only the measurements of J7B.
- In addition, we have the 2327 pass/fail tests that insure that both J7A and J7B were within the requirements. This count includes the 468 tests described above.

Likelihood

$$\prod_{i=1}^{383} \frac{1}{\sigma_A} \exp(-\frac{1}{2\sigma_A^2}(x_i - \mu_A)^2)$$
$$\prod_{j=1}^{468} \frac{1}{\sigma_B} \exp(-\frac{1}{2\sigma_B^2}(y_i - \mu_B)^2)$$
$$((\Phi(4.5, \mu_A, \sigma_A) - \Phi(-4.5, \mu_A, \sigma_A))$$
$$(\Phi(3.8, \mu_B, \sigma_B) - \Phi(-3.8, \mu_B, \sigma_B)))^{1859}$$

Posterior



Density
0
1000
2000
3000
4000
5000
0.9990
0.9992
0.9994
0.9996
0.9998
1.0000
P(J7A and J7B)

Assurance Testing Example

- Six previous systems with similar components
- Each previous system had 20 tests done
- Temperature ranges from 30° to 70°
- Chemistry parameter ranges from 0 to 1
- Approximately 15% of the data is right-censored
- Simulated data

More on the Example

- Interest in developing a replacement component
- "Life extension program"
- Two ways to think about lifetime
  - Time since built
  - Time in use
- Six previous systems with similar components
- Temperature and "chemistry parameter" predictive of lifetime

Reliability Demonstration and Assurance

- Example: Using minimal assumptions, to demonstrate that reliability at time t0 hours is .99, with 90% confidence, requires testing at least 230 units for t0 hours with zero failures. To have a 80% chance of passing the test, requires that the true reliability be approximately .999.
- For complicated, expensive systems, traditional reliability demonstration is usually not practical
- Reliability assurance is the alternative: Use whatever relevant knowledge you have in a principled Bayesian approach to plan the test

What Test Plan?

Denote the test plan (n, t_0, c).

- How many of the new components (n) should I test?
- For how long (t_0)?
- How many can fail (c)?

Test Criteria

Interest centers on $t = 1$

- Posterior Producer's Risk: Choose a test plan so that if the test is failed, there is a small probability that the reliability at $t = 1$ is high
- Posterior Consumer's Risk: Choose a test plan so that if the test is passed, there is a small probability that the reliability at $t = 1$ is low
  - Reliable Life Criterion: Choose a test plan so that if the test is passed, there is a high probability that the $1 - \alpha$ quantile of the distribution is greater than $t = 1$

Model

- Fit the data using a Weibull regression
- Common shape parameter β
- Scale parameter λ_{ij}
  - $i = 1, \ldots, 6$, $j = 1, \ldots, 20$
  - $\log(\lambda_{ij}) = \gamma_0 + \gamma_1 \text{Temp}_{ij} + \gamma_2 \text{Chem}_{ij} + \gamma_3 (\text{Temp x Chem})_{ij} + \omega_i$
  - Common regression model with hierarchical "random effect" ω_i for each system
- $\omega_i \sim N(0, \tau_\omega)$
- Diffuse priors for $\gamma_0, \gamma_1, \gamma_2, \gamma_3, \tau_\omega, \beta$

More on the Example

- Assume the new system follows the common regression model with its own parameter ω_N
- Choose a test plan (n, t_0, c) so that given the previous data, if the test is passed, there is a 0.9 probability that the 0.05 quantile of the distribution when $\text{Temp} = 50^\circ$ and $\text{Chem} = 0.5$ is greater than $t = 1$
- Choose $c = 0$ so that the test is passed only if there are no failures
- $q_{0.05} = \lambda^{-1/\beta}[-\log(0.95)]^{1/\beta}$

Computation

$$\mathbf{P}(q_{0.05} > t_{R^*} = 1 \mid TIP, \mathbf{t})$$

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$$\mathbf{P}(q_{0.05} > t_{R^*} = 1 \mid TIP, \mathbf{t})$$
$$= \mathbf{P}(\lambda < -\log(0.95) t_{R^*}^{-\beta} \mid TIP, \mathbf{t})$$

Computation

$$
\begin{aligned}
&\mathbf{P}(q_{0.05} > t_{R^*} = 1 \mid TIP, \mathbf{t}) \\
&\quad = \mathbf{P}(\lambda < -\log(0.95) t_{R^*}^{-\beta} \mid TIP, \mathbf{t}) \\
&\quad = \int_0^{\infty} \int_0^{-\log(0.95) t_{R^*}^{-\beta}} f(\lambda, \beta \mid TIP, \mathbf{t}) \, \mathrm{d}\lambda \mathrm{d}\beta
\end{aligned}
$$

Computation

$$
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&\quad = \int_0^\infty \int_0^{-\log(0.95) t_{R^*}^{-\beta}} \frac{\mathbf{P}(TIP \mid \lambda, \beta, \mathbf{t}) p(\lambda, \beta \mid \mathbf{t})}{\int_0^\infty \int_0^\infty \mathbf{P}(TIP \mid \lambda, \beta, \mathbf{t}) p(\lambda, \beta \mid \mathbf{t}) \mathrm{d}\lambda \mathrm{d}\beta} \, \mathrm{d}\lambda \mathrm{d}\beta
\end{aligned}
$$

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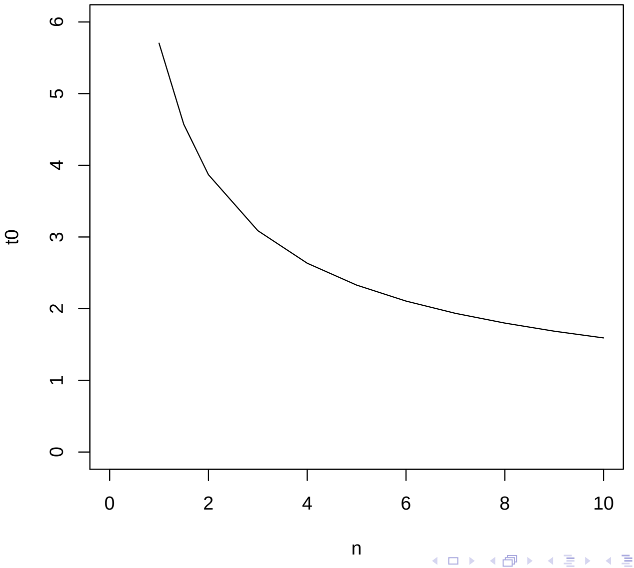
&\quad = \frac{\int_0^\infty \int_0^{-\log(0.95) t_{R^*}^{-\beta}} \exp(-n\lambda t_0^\beta) p(\lambda, \beta \mid \mathbf{t}) \mathrm{d}\lambda \mathrm{d}\beta}{\int_0^\infty \int_0^\infty \exp(-n\lambda t_0^\beta) p(\lambda, \beta \mid \mathbf{t}) \mathrm{d}\lambda \mathrm{d}\beta} \geq 0.9
\end{aligned}
$$

Monte Carlo Approximation

$$
\begin{aligned}
&\mathbf{P}(q_{0.05} > 1 \mid TIP, \mathbf{t}) \\
&\quad = \frac{\int_0^\infty \int_0^{-\log(0.95)} \exp(-n\lambda t_0^\beta) p(\lambda, \beta \mid \mathbf{t}) \mathrm{d}\lambda \mathrm{d}\beta}{\int_0^\infty \int_0^\infty \exp(-n\lambda t_0^\beta) p(\lambda, \beta \mid \mathbf{t}) \mathrm{d}\lambda \mathrm{d}\beta} \\
&\quad \approx \frac{\sum_{j=0}^N \exp(-n\lambda^{(j)} t_0^{\beta^{(j)}}) I[\lambda^{(j)} \leq -\log(0.95)]}{\sum_{j=0}^N \exp(-n\lambda^{(j)} t_0^{\beta^{(j)}})}
\end{aligned}
$$

For each n, solve for t_0.

Weibull Assurance Test Plans



Observations

- Straightforward generalization to $c > 0$ test plans
- Similar framework applicable to other test situations: for example, pass/fail or failure count data

Conclusions

- Science-Based Stockpile Stewardship is a rich source of statistical problems, requiring methodological development in specific problem contexts
- Example of simultaneous inference within a system reliability representation using multilevel data
- Reliability assurance methodology provides a formal way to use prior information to develop executable test plans
- Extension to "resource allocation" for entire system

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