The Role of the “PD” in Today’s Banking System
Takeaways

- A “PD” is playing an increasingly central role in today’s banking system

- A PD’s meaning can differ, depending on context
  - An internal rating
  - The long run average of the one-year default rate of a specific population within a specific risk profile
  - The best estimate of the PD given available information and a model

- Level Validation of a “PD” is challenging
  - Correct models will overstate default rates most of the time
  - There are many data issues associated with how to define default
  - Data collection methods are not constant over long time periods
  - The level of a PD needs to be benchmarked against other sources

- Reasonable differences in methodologies can lead to large differences in results
How did we get here?
In 1999 two sub-prime banks failed, The National Bank of Keystone (Keystone) and Pacific Thrift and Loan (PTL). Keystone was supposedly a $1.1 billion-bank that specialised in subprime lending. Keystone securitised some of these loans and had large concentrations of “retained interests” (RIs). The RIs were the primary concern of the supervisory authority and the proper valuation of the RIs was the principal source of disagreement between the regulators and the institution. When regulators began an examination of the institution in 1999, fraud was detected. Keystone had booked hundreds of millions of loans that did not exist or were not owned by Keystone. The FDIC resolution costs were close to $780 million 71% of reported assets.

Traditional Middle Market Banking

Reference Portfolio

Loans to Small and Medium Size Enterprises

Liabilities

92% Senior Tranche (depositors)

8% Equity Tranche (bank shareholders)

- Was relationship based
- Small spread earned on a levered portfolio
- If losses and expenses were low, then return on equity could be substantial
There is a Clear Need for Risk-Sensitive Capital Requirements

- “Under the risk-based capital rules (of Basel I), the most capital that a bank was required to hold was 8% of the principal of the loan. This was meant to equate to the risk of a standard C&I loan. Sub-prime lending had a much higher risk profile than a C&I loan, but had the same risk-based capital charge. By engaging in sub-prime lending, an institution could be in full compliance with all capital rules but in reality be operating with greatly increased leverage.”

Under Basel II, Regulatory Capital is Largely Determined by the PD

\[ K = EAD \times LGD \left( \Phi \left( \frac{\Phi^{-1}(PD_i) + \sqrt{\rho_i} \Phi^{-1}(0.999)}{\sqrt{1-\rho_i}} \right) - PD \right) \]

K is capital requirements
EAD is exposure at default
LGD is loss given default
PD is the one year probability of default
\( \rho_i \) is “asset correlation”
0.999 sets the capital requirement to the 1 in a thousand worst case scenario
This is Vasicek’s limiting distribution and assumes a highly diversified portfolio and a one-factor model
Risk Weights Depend on the PD

Illustrative IRB Risk Weights for UL

<table>
<thead>
<tr>
<th>Asset Class:</th>
<th>Corporate Exposures</th>
<th>Residential Mortgages</th>
<th>Other Retail Exposures</th>
<th>Qualifying Revolving Retail Exposures</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>45%</td>
<td>45%</td>
<td>45%</td>
<td>85%</td>
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<tr>
<td></td>
<td>25%</td>
<td></td>
<td>25%</td>
<td></td>
</tr>
<tr>
<td>LGD:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maturity:</td>
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<td></td>
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</tr>
<tr>
<td>2.5 years</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Turnover</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(millions of €)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PD:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.03%</td>
<td>14.44%</td>
<td>11.30%</td>
<td>4.15%</td>
<td>2.30%</td>
</tr>
<tr>
<td>0.05%</td>
<td>19.65%</td>
<td>15.39%</td>
<td>6.23%</td>
<td>3.46%</td>
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<tr>
<td>0.10%</td>
<td>29.65%</td>
<td>23.30%</td>
<td>10.69%</td>
<td>5.94%</td>
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<tr>
<td>0.25%</td>
<td>49.47%</td>
<td>39.01%</td>
<td>21.30%</td>
<td>11.83%</td>
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<tr>
<td>0.40%</td>
<td>62.72%</td>
<td>49.49%</td>
<td>29.94%</td>
<td>16.64%</td>
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<tr>
<td>0.50%</td>
<td>69.81%</td>
<td>54.91%</td>
<td>35.08%</td>
<td>19.49%</td>
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<tr>
<td>0.75%</td>
<td>82.76%</td>
<td>65.14%</td>
<td>46.46%</td>
<td>26.81%</td>
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<tr>
<td>1.00%</td>
<td>92.32%</td>
<td>72.40%</td>
<td>59.40%</td>
<td>31.33%</td>
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<tr>
<td>1.30%</td>
<td>100.55%</td>
<td>76.77%</td>
<td>67.00%</td>
<td>37.22%</td>
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<tr>
<td>1.50%</td>
<td>105.59%</td>
<td>82.11%</td>
<td>73.45%</td>
<td>40.80%</td>
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<tr>
<td>2.00%</td>
<td>114.86%</td>
<td>88.55%</td>
<td>87.94%</td>
<td>48.85%</td>
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<tr>
<td>2.50%</td>
<td>122.16%</td>
<td>93.43%</td>
<td>100.64%</td>
<td>55.91%</td>
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<tr>
<td>3.00%</td>
<td>128.44%</td>
<td>97.58%</td>
<td>111.99%</td>
<td>62.22%</td>
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<tr>
<td>4.00%</td>
<td>139.58%</td>
<td>105.04%</td>
<td>131.63%</td>
<td>73.13%</td>
</tr>
<tr>
<td>5.00%</td>
<td>149.66%</td>
<td>112.27%</td>
<td>149.22%</td>
<td>82.36%</td>
</tr>
<tr>
<td>6.00%</td>
<td>159.61%</td>
<td>119.49%</td>
<td>162.52%</td>
<td>90.29%</td>
</tr>
<tr>
<td>10.00%</td>
<td>193.09%</td>
<td>146.51%</td>
<td>204.41%</td>
<td>113.56%</td>
</tr>
<tr>
<td>15.00%</td>
<td>221.54%</td>
<td>171.91%</td>
<td>235.72%</td>
<td>130.96%</td>
</tr>
<tr>
<td>20.00%</td>
<td>238.23%</td>
<td>188.42%</td>
<td>253.12%</td>
<td>140.12%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>100.28%</td>
<td>189.41%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>117.99%</td>
<td>222.86%</td>
</tr>
</tbody>
</table>

page 197 of “A Revised Framework.”
## Different PDs Yield Different Capital Requirements

<table>
<thead>
<tr>
<th>PD (%)</th>
<th>Risk Weight (%)</th>
<th>Capital Requirements (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03</td>
<td>14</td>
<td>1.2</td>
</tr>
<tr>
<td>1.30</td>
<td>100</td>
<td>8.0</td>
</tr>
<tr>
<td>10.00</td>
<td>193</td>
<td>15.5</td>
</tr>
</tbody>
</table>

Based on an exposure with 50mm Euros of turnover; maturity, LGD and EAD are 2.5, 45% and 100%, respectively. See for example: page 197 of “A Revised Framework.”
What is a PD?
What is a PD?

- Is it: The probability of default within one-year given all information available and a model
  - How is default defined?
  - Is default a black and white event?

- Is it: The “long-run average” one-year default rate of a rating class?

- Is it: The output of an internal rating system used to determine:
  - Capital requirements
  - The level of monitoring
  - Loan limits
  - Pricing terms
  - Whether or not to originate? or Renew?
Within Rating Categories, PDs (as measured by the Moody’s KMV Public Firm Model) Vary Widely Over the ‘Cycle’

One Year EDF

Median values for the respective rating categories
Based on North American Non-Financial Firms
Source: Moody’s KMV Public Firm Model, as of December 31, 2008
3 Challenges in Measuring a Realized Default Rate
How Does One Measure a ‘Realized Default Frequency’

- First step is to measure the default rate:

\[
DF_t = \frac{D_t}{N_t} \quad \text{or} \quad DF_t = \frac{D_t}{N_t - W_t/2}
\]

Where \( DF_t \) is the default frequency, \( D_t \) is the number of defaults during the period, \( N_t \) is the number of obligors in the portfolio at the beginning of the period and \( W_t \) is the number of obligors withdrawn from the portfolio during the period.

There are different concepts as to what a firm is:

- Moody’s Investors Service uses the legal entity in their default studies
- Moody’s KMV Public Firm Model is calibrated to the corporate family

It is often difficult to apply a common definition of default

It is often difficult to know what \( N \) is and \( W \) is often difficult to track.
Basel Definition of Default

- *Paragraph 452 & 453*

- unlikely to pay
- past due more than 90 days on any material credit obligation to the banking group.
- non-accrued status.
- charge-off
- sells the credit obligation at a material credit-related economic loss
- bankruptcy

Source: “A Revised Framework”
### Operationalizing a Definition of Default from Loan Accounting System Data

<table>
<thead>
<tr>
<th>Default Events</th>
<th>Default Path #1</th>
<th>Default Path #2</th>
<th>Default Path #3</th>
<th>Default Path #4</th>
<th>Non Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>Substandard</td>
<td>90 DPD</td>
<td>90 DPD</td>
<td>Substandard</td>
<td>90DPD</td>
</tr>
<tr>
<td>Second</td>
<td>90 DPD</td>
<td>Substandard</td>
<td>Substandard</td>
<td>90 DPD</td>
<td>Pass Grade</td>
</tr>
<tr>
<td>Third</td>
<td>Non-Accrual</td>
<td>Non-Accrual</td>
<td>Non-Accrual</td>
<td>Pass Grade</td>
<td></td>
</tr>
<tr>
<td>Fourth</td>
<td>Charge-Off</td>
<td>Charge-Off</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Defaults**
Evaluating the level of the PD for Small and Medium Sized Enterprises
Level Validation of Private Firm Default Prediction Models has Been Challenging

- Typically based on financial statements and default information drawn from separate sources
- Default information is often based either on bankruptcy or reconstructed after the fact and, hence, incomplete
- Financial statements may not actually have debt outstanding associated with them
- Models are calibrated to a ‘Central Default Tendency’
For Any Given Bucket, the Realized Default Rate on a Portfolio has a Highly Skewed Distribution

- The loss distribution of a portfolio depends on the PD, EAD, & LGD of each exposures, the size of the exposures, and credit migration.
- The distribution of defaults would equal the loss distribution of a portfolio for which all the firms had an EAD of 1, an LGD of 100% and a maturity of 1 year.

**FIGURE 2** Distribution of defaults.

Distribution of possible default rates in one year for 1,000 exposures with a correlation of 0.2 and a PD of 1%.

- Implication: Most of the time a correct model will over predict defaults!
The Role of the “PD” in today’s banking system

Interpreting the analytical outputs

• Analysis is based on a 1-factor Gaussian model
  • Given a correlation assumption, the actual default rate can be compared to the predicted median p10, p90 as well as the average PD.
  • Given the actual default rate, the posterior distribution for the aggregate shock can be derived. One can also compute the P-value of the actual default rate, which is the probability of observing a default at or lower than the actual default rate.

The actual default rate

80% of the time the actual default rate should lie within the 10th and 90th percentiles

The average PD. Most of the time the actual default rate should be below the average PD.

The median predicted default rate. 50% of the time the actual default rate should be above (or below) the median
Interpreting the analytical outputs (Continued)

- Analysis is based on a 1-factor Gaussian model.
- Given a correlation assumption, the actual default rate can be compared to the predicted median, p10, p90 as well as the average PD.
- Given the actual default rate, the posterior distribution for the aggregate shock can be derived. One can also compute the P-value of the actual default rate, which is the probability of observing a default at or lower than the actual default rate.

P-value measures the probability of observing a default rate at or lower than the actual default rate.

Median value of the aggregate shock given the actual default rate.
Assume Rho=0

The Role of the "PD" in today's banking system
Assume Rho=0.05
The Role of the “PD” in today’s banking system

Recent Level Validation of Moody’s KMV RiskCalc US v3.1

- Based on Loan Accounting System Data of multiple US banks (160,000 financial statements, 60,000 firms, 2,000 defaults)
- Restricts to ‘Active Borrowers’
- Model adjusts for the ‘Credit Cycle’
- Actual Default is compared to the distribution implied by the model and a single factor Gaussian model with a $\rho=0.2$

Range predicted by model

Actual Default Rate
Observed Default Rate can be Compared to Charge-Offs and Delinquency Rates

Thru second quarter of 2008
Commercial and Industrial Loans
Charge-Offs implied PD is computed assuming an LGD of 40%
Conclusion
Conclusion

- The loss distribution of a portfolio depends on the PD, EAD, LGD and size of the exposures in it as well as the number of exposures in the portfolio and the degree of systematic risk between them, as well as other risk factors.

- Avoiding bank failures will require better measurement of all these portfolio features.

- We may have made the most progress with respect to the determining the PD, but it is still challenging.

- We are using the term PD in multiple ways which is creating some confusion.
Further Reading

Arora, Navneet, Jeffery Bohn, and Irina Korabiev, “Power and Level Validation of the EDF™ Credit Measure in the U.S. Market,” Moody’s KMV, 2005a.


Kurbat, Matthew and Irina Korabiev, (2002) “Methodology for Testing the Level of the EDF™ Credit Measure,” Moody’s KMV.