Bayesian inference for population prediction of individuals without health insurance in Florida

Neung Soo Ha[1]

[1]NISS

Outline

- Motivation
- Description of the Behavioral Risk Factor Surveillance System, BRFSS
- Description of posterior predictive distribution under informative/non-informative sampling
- Selection bias
- Inference for proportions of individuals without the health insurance
- Variation of maps

Motivation:

Interest:

1. Inference for the county-level proportions of persons without health insurance in FL, using the 2010 Behavioral Risk Factor Surveillance System, BRFSS.
2. Display the variation of maps

2010 BRFSS: Survey description

- The largest on-going telephone based survey; administered by the Centers for Disease Control and Prevention.
- Collect data on individual risk behaviors and preventive health practices for the adult population(18 years of age and older); collects state-specific data
  - ex: alcohol/cigarette consumption, general health status, health insurance,
- Uses a disproportionate stratified sample (DSS) design.
- In FL, 63 out of 67 counties were sampled.
- No cell phones for 2010.

Examples of observed data

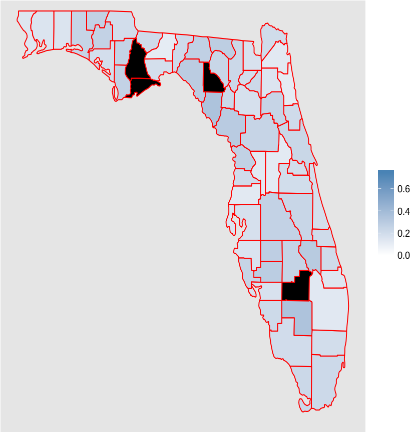


Figure: Observed: All races

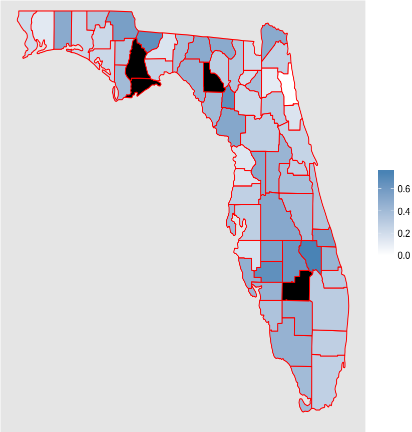


Figure: Observed: Hispanics

white=having insurance,
blue=not having insurance

Complication

1. Small or non-existent number of observations for some sub-population groups in certain geographical areas.
2. Possible presence of selection bias in the sample

Model specification for the BRFSS data

$$P(Y_{ik\ell} = 1|\theta_{ik}) = \theta_{ik}$$
$$\text{logit}(\theta_{ik}) = \mathbf{X}'_k \boldsymbol{\beta} + \nu_i$$
$$\nu_i \sim N(0, \sigma_\nu^2)$$

- $Y_{ik\ell} = 1$: not having insurance, $Y_{ik\ell} = 0$: having insurance
- $i = 1, \ldots, M$: county
- $k = 1, \ldots, K$: population class (9 age groups $\times$ 3 races $\times$ 2 genders)
- $\ell = 1, \ldots, N_{ik}$: units in county i and group k
- N_{ik} = population size in ith county and kth group
- $\mathbf{X}'_k$ = vector of indicators for group k.
- $p(\boldsymbol{\beta})$ = const on $(-\infty, \infty)$ Gelman et al.(2004)
- $p(\sigma_\nu^2)$ = const on $(0, \infty)$

Model Evaluation

Model fit

- Bayes residual plots
- QQ plots
- Bayesian tests: Partial posterior predictive p-values, Bayarri and Berger(2002)

Predictions

- Cross-validation to simulate finite population inference
  - Use 100(1-p)% of observed data to make predictions for the 100*p% that were "held-out"

Posterior predictive distribution with non-informative sampling

Use posterior predictive distribution to make inference for the remaining non-sampled units
Under the non-informative sampling, i.e. independence between the probability of selecting a person and the response:

- $\mathbf{Y}_s$ = sampled units, $\mathbf{Y}_{ns}$ = non-sampled units
- $\mathbf{Y}_{ns} \perp\!\!\!\perp \mathbf{Y}_s | \boldsymbol{\theta}, \mathbf{X}$
- $f(\mathbf{Y}_{ns}|\mathbf{Y}_s, \mathbf{X}) = \int g(\mathbf{Y}_{ns}|\mathbf{Y}_s, \mathbf{X}, \boldsymbol{\theta}) p(\boldsymbol{\theta}|\mathbf{Y}_s, \mathbf{X}) d\boldsymbol{\theta}$
  $= \int g(\mathbf{Y}_{ns}|\boldsymbol{\theta}, \mathbf{X}) p(\boldsymbol{\theta}|\mathbf{Y}_s, \mathbf{X}) d\boldsymbol{\theta}$
- $p(\boldsymbol{\theta}|\mathbf{Y}_s, \mathbf{X}) \propto L(\boldsymbol{\theta}|\mathbf{Y_s}, \mathbf{X}) p(\boldsymbol{\theta}|\boldsymbol{\beta}, \sigma_\nu^2) p(\boldsymbol{\beta}, \sigma_\nu^2)$: posterior distribution
  - $\boldsymbol{\theta}$ = a vector of model parameters
  - $(\boldsymbol{\beta}, \sigma_\nu^2)$ = a vector of hyperparameters

Under informative sampling

- Selection probabilities are related to the outcome variables.
- The observed outcomes may not be representative of the population outcomes.
- May cause bias for inferences about the finite population parameters.

Selection bias

Let $\mathbf{I} = (I_1, \ldots, I_N)$,
$I_i = 1$ if $i \in s$, $I_i = 0$ if $i \notin s$, $s =$ sample
The posterior predictive distribution with $\mathbf{I}$

- $$f(\mathbf{Y}_{ns}|\mathbf{Y}_s, \mathbf{X}, \boldsymbol{\theta}, \mathbf{I}) \propto g(\mathbf{Y}_{ns}|\boldsymbol{\theta}, \mathbf{X})p(\mathbf{I}|\mathbf{Y}, \mathbf{X}, \boldsymbol{\theta}) \tag{1}$$

Note:

- $p(\mathbf{I}|\mathbf{X}, \mathbf{Y}, \boldsymbol{\theta}) \neq p(\mathbf{I}|\mathbf{X}, \boldsymbol{\theta})$ i.e. selection bias.
- Selection information only comes from the sampled units.

Procedure: Ha and Sedransk(2015)

1. Pfeffermann and Sverchkov (2009):
   $p(\mathbf{I}|\mathbf{Y},\mathbf{X},\boldsymbol{\theta}) = p(\mathbf{I}|\mathbf{Y},\mathbf{X}) = \prod_k f_1(\mathbf{I}_k|\mathbf{Y},\mathbf{X_k})$
2. Using the Poisson sampling approximation, Chambers et. al.(1998)
   $p(\mathbf{I}|\mathbf{Y},\mathbf{X})$

$$= \prod_{i=1}^{M}\prod_{k=1}^{K}\{\prod_{\ell\in s_{ik}} P(I_{ik\ell}=1|\mathbf{Y},\mathbf{X}_k)\}\{\prod_{\ell\notin s_{ik}}(1-P(I_{ik\ell}=1|\mathbf{Y},\mathbf{X}_k))\}$$

3. $P(I_{ik\ell}=1|\mathbf{Y},\mathbf{X}_k) = E_U(\pi_{ik\ell}|\mathbf{Y},\mathbf{X}_k) = \{E_s(w_{ik\ell}|\mathbf{Y},\mathbf{X}_k)\}^{-1}$,
   Pfeffermann and Sverchkov (2009),
   - $\pi_{ik\ell}$ = probability of unit selected
   - $w_{ik\ell} = 1/\pi_{ik\ell}$

Posterior predictive distribution with inclusion probability

- $g(\mathbf{Y}_{ns}|\mathbf{Y}_s, \mathbf{X}, \boldsymbol{\theta}, \mathbf{I}) \approx$
  $g(\mathbf{Y}_{ns}|\theta, \mathbf{X}) \times \prod_{i=1}^{M} \prod_{k=1}^{K} (1 - a_k)^{M_{ik1}} (1 - b_k)^{M_{ik0}}$,
  - $a_k = \frac{1}{\bar{w}_{k,y0}}, b_k = \frac{1}{\bar{w}_{k,y1}}$
  - $\bar{w}_{k,y0}$ = average of inverse probability of selected units for class k with response $Y_{ik\ell} = 0$
  - $\bar{w}_{k,y1}$ = average of inverse probability of selected units for class k with response $Y_{ik\ell} = 1$
  - M_{ik1} : # non-sampled units with $Y_{ik\ell} = 1$
  - M_{ik0} : # non-sampled units with $Y_{ik\ell} = 0$
  - $M_{ik1} + M_{ik0} = N_{ik} - n_{ik}$
- Use rejection sampling algorithm to obtain $g(\mathbf{Y}_{ns}|\cdot)$, Robert, C. and Casella, G. (2004)

In our data set, $0.96 \leq (1 - a_k)/(1 - b_k) \leq 1.01$ with 43 of the 54 K groups.

- *No substantial selection bias.*

Comparison between prediction to observation

County comparison

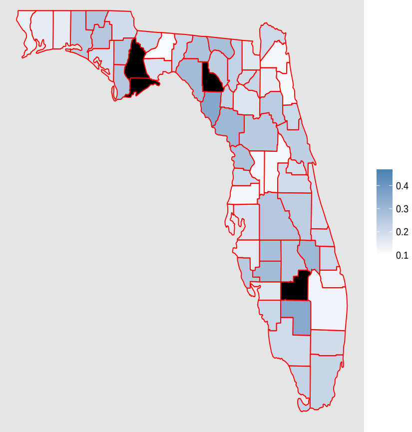


Figure: Observed: All races

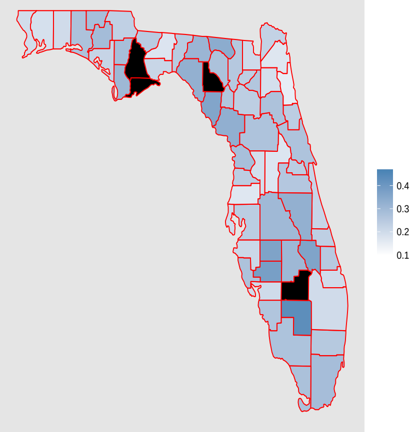


Figure: Predicted: All races

Comparison between prediction to observation

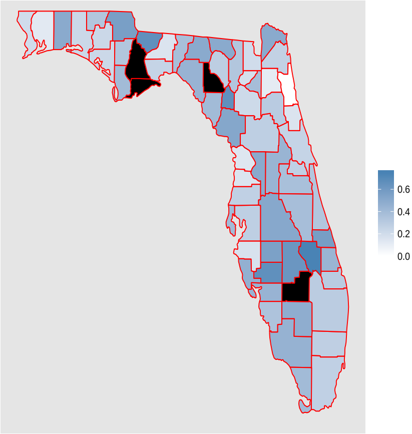


Figure: Observed: Hispanics

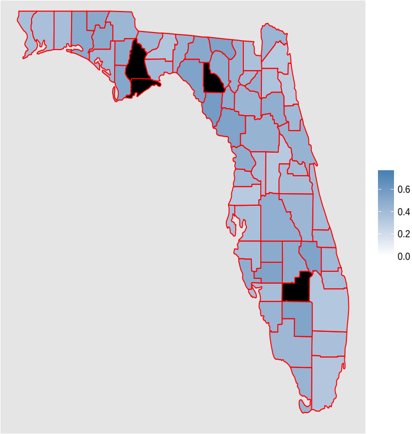


Figure: Predicted: Hispanics

Comparison between races

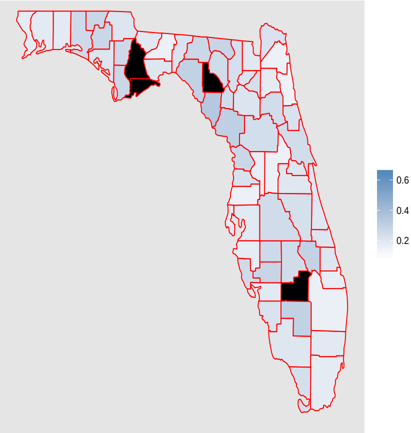


Figure: Predicted: Whites

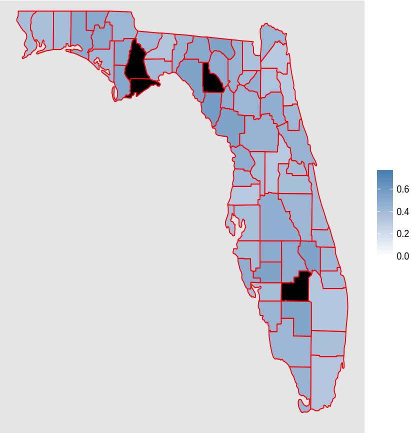


Figure: Predicted: Hispanics

Variation of maps

Objective: Assess the variation of the entire map as a unit rather than expressing the variation separately for each geographic unit.

- Produce a map, based on each MCMC replicate.
- Analyze variations from a set of maps

Illustrate variation between the counties from ν_i, the county-level random effect.

Mean choropleth map of random effect

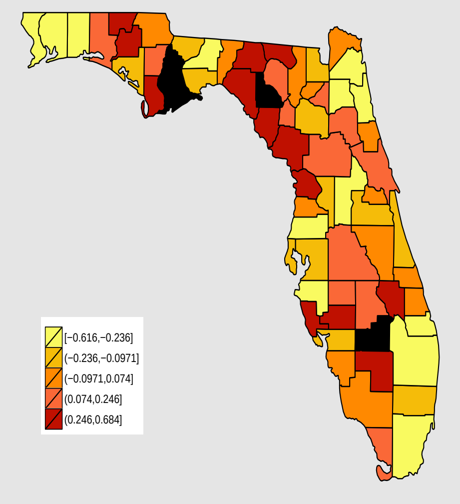


Figure: Mean map of $\hat{\nu}_i$

- Posterior means of county-level random effects, ν_i, are partitioned into quintiles
- Each quintile is color coded

Besag Method, Besag et al (1995)

- Provide $100(1-a)\%$ regions for each of ν_i: $\nu_i(L) < \nu_i < \nu_i(U)$
- Construct lower choropleth map of $\nu_i(L)$ and $\nu_i(U)$

1. Denote the posterior MCMC sample by $\{\nu_i^{(t)} : i = 1, \dots, M, t = 1, \dots, T\}$
2. Order $\{\nu_i^{(t)} : t = 1, \dots, T\}$ separately for each i to obtain order statistics $x_i^{[t]}$ and ranks $r_i^{(t)}$
3. For fixed $k \in \{1, \dots, T\}$, let t^* be the smallest integer such that $x_i^{[T+1-t^*]} \leq x_i^{(t)} \leq x_i^{[t^*]}$, for all i, for at least k values of t
4. $t^* = k$th order statistic from the set $a^{(t)} = max\{max_i r_i^{(t)}, T+1-min_i r_i^{(t)}\}, t = 1, \dots, T$, i.e. $t^* = a^{[k]}$
5. Then, $\{[x_i^{[T+1-t^*]}, x_i^{[t^*]}] : i = 1, \dots, M\}$ are a set of simultaneous credible regions containing at least $100k/T\%$ of the distribution

Lower and Upper maps

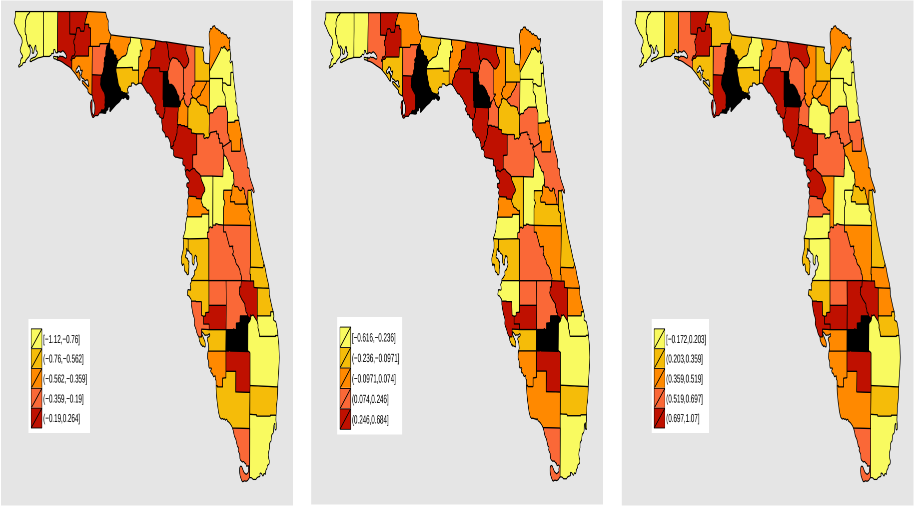


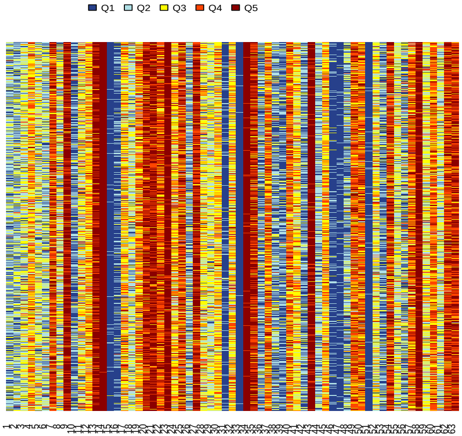
Figure: 90 % Lower, Mean, 90% Upper

Variation in Choropleth map

Goal: Illustrate the uncertainty of the choropleth maps.

- Use $T = 1000$ MCMC replications of ν_i
- Each replication can be used to produce a choropleth map.
- Each quintile is color coded.
- For each replication, the value of ith county is color-coded according to its quintile.

Heat map



- Rows: replicates, columns: counties
- Variation in maps is evaluated by color changes

Summary

- Present a method for posterior predictive inference that includes selection information for sampled units
- Illustrate variation of maps

Reference

1. Chambers, R. L., Dorfman, A. and Wang, W. (1998). Limited Information Likelihood Analysis of Survey Data. *Journal of the Royal Statistical Society*, **60**, 397-411
   Bayesian benchmarking with application to small area estimation *Test* **20** 574-588
2. Gelman, A., Carlin, J. Stern, H. and Rubin, D. (2004), *Bayesian Data Analysis*. Chapman and Hall/ CRC, New York, NY.
3. Pfeffermann, D. and Sverchkov, M. (2009) Inference under Informative Sampling *Sample Surveys: Inference and Analysis*, **29B**, 455-487
4. Robert, C. and Casella, G. (2004), *Monte Carlo Statistical Methods*, Springer, New York, NY.