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Abstract

In modeling trip-chaining behavior a number of issues arise the most fundamental being the underlying decision process. This issue is critical because it affects the modeling effort directly. In this paper we argue that trip-chaining behavior is based neither on a simultaneous nor on a sequential decision process, but on a rather in-between particular type of sequential decision making process called a *relatively-sequential* process. Moreover, single-stop trip chains seen in the light of *relatively-sequential* decision processes comprise the building blocks of complex trip chains. The *relatively-sequential* process is further formally characterized in the tradition of the probabilistic threshold theory.

The threshold theory offers an explicit behavioral interpretation of looking at activity sequencing as an r-sequential spatial interaction process. The modeling effort deals with individual attitudes toward trip chaining yielding macro frequencies, representable by general gravity models. These frequencies are shown to be filtered frequencies of more traditional gravity models for unchained trip flows. A family of threshold gravity models is proposed corresponding to individual commuters' attitudes toward separation. In this manner, this alternative modeling framework may serve to bridge the gap between macro and micro approaches in trip-chain models since it can use data typically gathered in transportation studies.

Maximum likelihood estimation of separation-threshold gravity models based on a home interview survey show that the proposed theoretical framework has initiated a legitimate and promising research direction. Indeed, estimated trip-chain frequencies fit very well to data. Given that estimation results are based on only few observations, the very good fit of the models is an indication that threshold gravity models may not be "data-hungry". This, in turn, give another evidence that threshold gravity models although aggregate in principle may not need more data than disaggregate models to study the distribution of trip-chain frequencies. An additional very significant empirical finding is that in the presence of the data available home-to-work and the reverse single-stop trip chains may be indistinguishable in the sense that there may be no need for separate parameter estimates. This is a very pleasant result from the transportation modeler's perspective. Finally, short-term forecasting suggestions are made under reasonable assumptions and analytic relationships for forecast trip-chain frequencies are proposed along with a method to compute them.

1 Introduction

Locational shifts in the past may have reduced the growth of congestion in cities, but introduced congestion in the suburbs and beyond where most of the new job creation has been taking place. Congestion which used to be a problem for the urban inbound (in the morning) and outbound (in the evening) freeways, now has become a problem on major arterials for suburb to suburb commuting trips. Interestingly, large increases in travel delays are occurring although commuting trips are not getting longer on average. This could mean that most of the direct impacts of congestion are falling on nonwork trips for shopping, social and recreational purposes and other family or personal business which once avoided congestion simply by avoiding the traditional rush hour.

While the above are general observations on travel patterns taken from snapshots of periodic surveys, at the same time empirical evidence suggests that during the last twenty years, trip chains have become longer and substantially more complex and that the average number of trips per chain has increased. Consequently, the transportation modeler is faced not only with the burden to account convincingly for nonwork trips (traditionally the most difficult set of trip purposes to estimate), but also with the additional burden to represent work and nonwork trips as combined activities.

Let us offer a definition for trip chains and illustrate different aspects of trip-chaining behavior. A trip chain may be defined as a sequence of trips that starts from home and/or ends at home. This general definition is made more explicit via the illustration in Figure 1 of three different examples of trip chains. The first example presents a *simple trip chain*, namely a trip chain with only one stop for a particular activity, or a combined trip with two legs of the form home–destination–home. This type of trip chain does not include a nonhome-based trip (a trip either end of which is the home location) other than the return home trip. The other two examples show various manifestations of *complex* or *multistop trip chains*. The latter type of trip chains includes more than one stop before returning home. Therefore, in a complex trip chain, nonhome based trips are also considered.

Different taxonomies of complex trip chains are possible depending on the purpose or mode of the trip for different classes of travelers. The sequence of trips 1 to 9 in Figure 1, for example, can be thought to belong to a *shopping trip chain*, or to a *work trip chain* since work and shopping activities are both included. These trips may involve only the car mode or a series of modes (walk, carpool, transit, etc.); in the latter case we may also consider *mode-chaining* along with activity chaining. Moreover, these trips may be taken by the same person or different persons in the same or different households.

One implication of trip-chaining behavior is illustrated in Figure 2. The first

Figure 1: Hypothetical trip chain patterns for the same household

household engages in five different activities in the same trip chain taking six trips. The second household takes six trips for only three activities. When It would appear that household A behaves more efficiently than household B. This may be true at the individual household level. Bundling more trips in a single chain, however, lengthens the duration of travel and, for work trip chains, these trips add to the congestion in the morning and afternoon peak hour.

Different factors related to changes in the transportation system and the socioeconomic and demographic profiles of urban travelers have been found to induce changes in trip-chaining behavior. Location within the metropolitan region and the time of day are additional factors. Residents of areas closer to the central city are less likely to link work and nonwork activities compared with those living in the outer suburbs. Furthermore, commuters chain multiple activities more in the afternoon than in the morning.

These facts have triggered a shift of attention from individual trips to trip chains (for extensive reviews of the literature see Thill and Thomas, 1987; Kitamura, 1988; Metaxatos, 1995). It is generally accepted that travel is a derived activity taken to fulfil different goals. Consequently, activities which may or may not translate into trips (at least with a motorized mode) should be viewed as chained events. We will see however later in the discussion that our treatment of trip chains is amenable to such behavioral requirements. But first we will explain why current travel models cannot represent trip

Figure 2: Hypothetical trip chain patterns for different households

chains.

2 Inability of Current Travel Demand Models to Address Trip Chaining Behavior

Traditionally, in the four step procedure, travel is estimated separately for several purposes and the resulting trip groups are combined and assigned to the transportation system. Trips that begin or end at home (home-based trips) are treated separately from trips that do not begin or end at home (nonhome-based trips). The reason is that many of the factors which influence trip-making decisions are related to the characteristics of the traveler. Therefore, trips are usually estimated by using forecast characteristics of the traveler and the family or household from data about their place of residence, which frequently are the only data available.

Let us examine how a traditional gravity model (and for that matter any model based on the independence of irrelevant alternatives assumption) would treat an intermediate destination in a single-stop trip chain. Let $T_{ij} = A_i B_j \exp(\theta c_{ij})$ be a gravity model for the flows between destinations i and j. Assume, for example, that there are two shops located in zones, j and k, respectively, with identical attributes ($B_j = B_k$) at equal distances from the work location w ($c_{wj} = c_{wk}$). Assume, further, that the first shop is located in zone j, between home and work, while the second shop is located in zone k as shown in Figure 3. For the purpose of illustration, assume that $c_{jh} = 10$ minutes and $c_{kh} = 20$ minutes. The relative odds of visiting destination j as opposed to destination k would be then

$$\frac{P_{wj}}{P_{wk}} = \frac{A_w B_j \exp(\theta c_{wj})/T}{A_w B_k \exp(\theta c_{wk})/T} = \exp[\theta(c_{wj} - c_{wk})] = \exp(\theta \times 0) = 1.$$

where, T, the total number of trips. There is a greater likelihood, however, that a traveler on the way back to home after work will stop at location j than in location k. Therefore, if the returning home traveler stop to shop and the two trips are treated as two independent trips, there is nothing in the traditional gravity model that can prohibit the distribution of trips to location k. As a result, this location will be overestimated.

Figure 3: Competing intermediate destinations on the way from work to home.

The implications of trip-chaining behavior for transportation planning models are serious, affecting their formulation, estimation and application. Existing transportation planning models assume independence among the trips linked in a trip chain. As a result, home-based and nonhome-based trips can be easily clustered in two groups and studied separately. An increase, however, of trip-chaining behavior increases, given a constant total, the number of nonhome-based trips, or, equivalently, decreases the number of home-based trips. As a result, the former would be overpredicted and the latter underpredicted.

Home-based and nonhome-based trips are increasingly interdependent with respect to their length, duration and scheduling for a number of reasons, such as an increase in auto ownership, multiple-worker households, urban sprawl, etc. Different classifications that consider interdependency among successive trips in a trip chain are then needed. For example, one may consider *work trip chains* instead of work trips, or *shopping trip chains* instead of shopping trips and new models need to be built to attest, among others, to the observation that an increased number of trips bundled in trip chains reduce the number of home-based trips.

In modeling trip-chaining behavior a number of issues arise the most fundamental being the underlying decision process. This issue is critical because it affects the modeling effort directly. In this paper we argue that single-stop trip-chaining behavior is based neither on a simultaneous nor on a sequential (in the traditional sense) decision-making process , but on a rather in-between particular type of decision-making process called a *relatively-sequential* process. Moreover, we reason that single-stop trip chains seen in the light of *relatively-sequential* decision processes comprise the building blocks of complex trip chains.

The rest of the paper is organized as follows: In section 3 a discussion on the decision making process underlying trip chaining is presented. The *relatively-sequential* decision process is formalized in section 4. Empirical results are presented in section 5. Finally, concluding remarks are made in section 6.

3 Trip Chaining as a Decision-Making Process

Much of the discussion concerning the development of trip chaining models is devoted to what decision-making process is manifested when trips are bundled into chains. The issue is of fundamental importance because it determines the model building process. The complexity of trip chain patterns gives rise to *sequential* models in which a trip chain is decomposed into its component trips (Horowitz, 1978; Lerman, 1979; O'Kelly, 1981; Van der Hoorn, 1983; Kitamura, 1984; Kim, 1993). On the other hand, *simultaneous* models treat a trip chain as one multidimensional entity (Adler and Ben-Akiva, 1979; Narula *et al*, 1983; Bacon, 1984; Ingene, 1984; Borgers and Timmermans, 1986).

In the simultaneous approach the underlying assumption is that the attributes of all conceivable destinations, for all intermediate destinations are perceived and evaluated at the same time resulting in a particular activity schedule. In this context, an individual is assumed to maximize the benefits or minimize the disbenefits of his/her journey for different activities, given the time and/or money constraints, or even the activities of other members of the same household and the number of the alternative choice sets may be quite large. Simultaneous choice of the intermediate destinations would imply, for example, that work locations and gas stations are chosen at the same time. This, of course would not be usually true and could ignore the fact that the choice of a gas station as part of a work trip chain would frequently depend on the work location.

In the sequential approach the implication is that travelers decide where to travel next only after they reach the previous destination. Thus, the utilities of arriving at each destination are maximized sequentially at each stage. If the first destination is chosen, the next destination is based on an evaluation process in which trip makers trade off the locational and nonlocational attributes of potential destination alternatives. The process continues until all activities are exhausted. A computational advantage of the sequential approach, as compared with the simultaneous approach, is the reduction of the problem to a manageable size. The sequential approach may be of some value to modeling nonwork activity chains, although it does not provide further insights with respect to the activity sequencing. Moreover, to return to the previous example, sequential decision making would imply that if a person stops to buy gas on the way to work, the decision on the work location would be reached only after arriving at the gas station.

Travel decisions may be sequential to a certain extent, but we believe the sequence of decision making is different from the one that is commonly assumed. Mandatory trips, for instance, such as work or school trips can be scheduled in advance because the location of those activities is known. Discretionary trips, on the other hand, such as shopping trips, recreational trips, or eating-out trips are less planned and they may be scheduled just before the trip is made. The sequencing of both types of activities, however, is uncertain because their level of priority at the individual level is unknown (see also Pas and Subramanian, 1995; Kumar and Levinson, 1995). Thus, it would be arbitrary to consider mandatory and discretionary trips jointly in a traditional simultaneous or sequential framework.

If mandatory trips are planned long before they are made, while discretionary trips just before or shortly before, the questions at issue is how to accommodate both type of trips in the same decision-making process and what type of process this would be. For a possible answer we may look into the relative variability of the destinations visited. Let us consider, for example, the case in which trips for the same activities are made to the same destinations repeatedly. This behavior may be observed in situations involving visits to supermarkets conveniently located and having the "right" price, visits to restaurants with the preferred food and atmosphere, visits to the same friendly dry cleaner's, visits to the same friends, etc. In situations like these, the resulting traffic patterns can be determined with certainty. On the other hand, there may be situations where a more adventurous behavior is observed for the previous type of activities. In cases, for example, of young travelers, or newcomers in an area, or in cases of a more flexible behavior as a result of gender or income variations, or in cases during which a trip maker is faced with unexpected obstacles, such as inclement weather or poor road conditions the resulting traffic patterns need not be determined with the same level of certainty.

Residential and employment locations do not change as frequently as other destinations in the short run. Thus, they may be treated as fixed destinations in one's everyday trip schedule since trip makers do not have alternative choices at those destinations. Other choices for discretionary trips then can be based on those fixed choices and the overall variability may be small enough to support a simultaneous decision making process for the entire trip chain. There may be reasons to believe that home-to-work and work-to-home trip chains need a different treatment because these trip chains are scheduled at different times of day and time constraints allow for different levels of variation in the choice of intermediate destinations. Time constraints restrict the number of destination choices in the home-to-work trip chain, but may be relaxed considerably on the return home trip. In the last case, however, fatigue of the trip maker may compensate for the looseness of the time constraint. As a result, the variability of the intermediate destination choices may be lowered at the level of that in the home to work trip chain.

To provide arguments for a new concept of trip-chaining decision making, it is interesting to observe that both the established simultaneous and sequential approaches and the corresponding trip-chaining models imply that travelers behave as rational economic human beings. As such, those models demand more capabilities than travelers can deliver. Conversely, another description of travel behavior based on limited or bounded rationality could provide an alternative behavioral basis which would lead to largely unexplored modeling frameworks. An important theoretical support for a sequential (seen in a different context as explained below) trip-chaining decision making could come, for example, from a satisficing (Simon, 1957) rather than optimizing individual behavior.

Satisficing behavior rejects the idea that there exists a rational economic agent who is perfectly knowledgeable and perceptive about all the possible alternatives (here, destinations for each intermediate stop) under consideration. Thus, this agent would be in great difficulty to compare all possible alternatives with one another to find an optimal choice manipulating attributes describing the alternatives. Satisficing substitutes for this true or complete rationality a hypothesis of bounded rationality. This implies sequential search (although not necessarily any specific search process) and limited sets of criteria used for evaluation. That is, (see also Brand, 1974) instead of comparing one alternative to another on the basis of a set of operational criteria, the alternatives are compared to a simpler set of minimal criteria until an alternative is found that satisfies the decision maker. No attempt is made to exhaust all possible alternatives. Moreover, search for new alternatives will only occur if the traveler perceives a discrepancy between his/her level of aspiration and his/her level of reward from the existing behavior.

Interestingly, however, not only do we suggest that a traveler may sequentially apply a limited set of criteria that are used to reject alternatives that do not meet threshold levels of those criteria, but also that those evaluations may take place at different levels. To elaborate on this idea, we propose that what is manifested in the choice of intermediate destinations is a *relatively-sequential* (in the absence of a better terminology) decision making. Under this type of sequential behavior the choice of successive destinations is made in a certain order over time, but this order need have little to do with the order of the destinations in the trip chain. Work destinations would typically be chosen before shopping destinations and the latter would usually be affected by the former choice. Similarly, the choice of school destinations or a doctor's appointment would precede the choice of a gas station. Thus, in this approach, destinations are prioritized by the order in which the decision was taken or by the importance of the decision.

The new approach is exemplified in Figure 4 where we start with a high priority trip from 1 to 2, say the trip to work. On the way, if one needs to stop at the dry cleaner's, a suitable destination would be chosen which is convenient for the trip from 1 to 2. The situation would then be as depicted in Figure 4 where the trip from 1 to 2 is now via 3, i.e., we now have two trips, one from 1 to 3 and one from 3 to 2. Similarly, either leg 1 to 3 or 3 to 2 could be interrupted by another intermediate destination. Thus, modeling can proceed in steps. The choice of the highest priority destination would be modeled first. At the next step, the choice of an intermediate destination would be modeled. At the next step an intermediate destination on one of the previously determined trips would be considered. Therefore, multi-stop, multi-purpose activity-making may be represented as a process in which sequences of one-stop trip chains proceed in space. Thus, for modeling purposes it is enough to consider how a single intermediate destination or stop is chosen.

Figure 4: R-sequential decision making for intermediate trips

The behavioral implication of this new approach is that the terms "mandatory" and "discretionary" are no longer viewed as permanent (absolute) labels, bur rather relative attributes describing individual activities, which can be prioritized so that separate activities can be linked together. A trip made in response to a doctor's appointment, for example, is viewed in this sense as a discretionary activity as compared to the trip related with the activity of going to work, but mandatory (or less discretionary) with respect to a visit to a fast-food restaurant. This is the reason why we have called this decision process *relatively-sequential* or, abbreviating, *r-sequential* decision process.

In the next section, we operationalize the *r-sequential* trip-chaining decision-making process. Although the theoretical framework considered is not restricted to particular trip-chaining situations, and hence, shopping and work trip chains are equally amenable, we have chosen to focus on work trip chains for two reasons: a) modeling nonhome-based trips is still a challenging task in the current transportation modeling state-of-the-art; and b) errands for personal business are very often scheduled as part of work trip chains in the morning or afternoon peak adding to congestion problems.

4 Theoretical Framework

In order to operationalize an *r-sequential* decision-making process for single-stop work trip chains, we will develop a theoretical framework which treats these type of processes as spatial interaction processes. Let us assume that there exist a large population of commuters $\alpha \in A$, residing in zones $i \in I$, and working in zones $j \in J$. Let $W = \{(i,j) :\in I \times J\}$ be the product set of those origins (home or work) and final destinations (home or work) involved in a single-stop work trip chain. Similarly, assume a large population of opportunities for discretionary activity $\beta \in B$, distributed over a spatial configuration of intermediate destinations $k \in K$. The set of those work trip chains then may be described by the *spatial interaction pattern* $s = \{(w,k) : w \in W, k \in K\}$.

With each pair $(w,k) \in W \times K$ we associate a vector of relevant separation values designated as the *wk-separation profile* c_{wk}. Any measure of separation between a commuting trip w and an intermediate stop at k potentially considered by the individual commuter during his/her evaluation of the costs involved are elements of this vector. More precisely, $c_{wk} = (c_{wk}^{q} : q \in Q) \in R^{Q}$, where, for example, $c_{wk}^{(1)}$ = travel time, $c_{wk}^{(2)}$ = travel cost, $c_{wk}^{(3)}$ = car availability and so forth. We consider now all wk-separation profiles and designate this collection, $c = (c_{wk} : wk \in W \times K) \in R^{W \times K \times Q}$, as the *separation configuration* between W and K. Finally, C designates the set of all separation configurations c. All relevant spatial information is assumed to be specified by the choice of $c \in C$.

The choice of appropriate partitions of the travelers at the origin or final destination of the trip and opportunity populations in intermediate destinations with respect to spatial aggregation involves implicitly some measure of within-group homogeneity.

The choice, of course, is not unique. In the case of interval/ratio separation measures a number of aggregation functions exist such as the average value of all possible costs which may be involved in interactions between w and k. Alternatively, a standard clustering procedure would identify partitions which minimize the total within-group variance. In the more general case of ordinal separation measures the choice of meaningful aggregation functions is more limited. A possibility would be to measure homogeneity of traveler populations in terms of the rank correlations between their individual separation profiles with all opportunities in intermediate destinations.

An important question arises now. How different types of spatial separation between w and k influence the likelihood of trip-chaining behavior? Spatial separation may be defined in terms of measures which are spatial or aspatial; or in terms of measures of interval/ratio, ordinal or categorical type. For example, car availability, which clearly affects the possibility to trip-chain, is a categorical aspatial element of a separation profile. In addition, travel time, distance and cost are normally identified as spatial ratio elements. Moreover, preference for a particular service is an aspatial ordinal element. What is important from a modeling perspective is whether the elements of the vector of separation measures are quantifiable in terms of meaningful units of measurement.

While it is difficult to study the behavior of individual commuters in detail, we can still obtain a reasonable overall picture of interaction behavior in terms of appropriately stratified spatial aggregations as follows. First, observe that while individual travelers have different commuting patterns, all commuters with the same home-work route w are locationally very similar in terms of their accessibility to opportunities in intermediate destinations. More generally, we assume that there are many partitions of the commuting population A into finite collections $\{A_w : w \in W\}$ of subgroups of commuters, in which each subgroup is reasonably homogeneous in terms of accessibility to intermediate destinations. Similarly, with respect to intermediate destinations, one may assume that travel times to all opportunities on the same zone are essentially identical. Hence, we assume that there are also many partitions of the population B into finite collections $\{B_k : k \in K\}$ of spatial subpopulations (opportunity zones), in which each subpopulation is reasonably homogeneous in terms of accessibility to commuters.

Having partitioned the population of commuters and intermediate activities this way, we only require that those partitions are sufficiently small to ensure spatial homogeneity. In this manner, the spatial separation between commuters in zonal pair w and intermediate activities in zone k can be approximated by a single representative separation profile c_{wk} an element of which may be, for example, the average travel time between w and k.

Stopping at k involves a decision by commuter $\alpha \in A_w$ to satisfy his/her own need or desire to pursue a discretionary activity $\beta \in B_k$. We may assume, further, that a traveler's decision on whether or not to stop at k depends on his/her current willingness to incur all associated costs of deviating at k (extra travel time plus time spent during the activity, extra costs, extra effort, etc.). Each individual commuter then has a particular tolerance toward overcoming the spatial separation between his/her mandatory direct route and the indirect route involving a discretionary activity. This tolerance, of course, varies with the time of day (different time constraints in the morning than in the afternoon), individual socioeconomic characteristics (age, gender, marital status, car availability, flexible work schedule/full time employment), familiarity with the area, taste and preferences, weather conditions and so forth. Individual variability in tolerance levels toward stopping at intermediate destinations, especially on a daily basis, can be higher than variability across individuals, particularly if those commuters belong to relatively homogeneous groups.

Since substitution (tradeoff) effects among different separation measures will not be considered here (at least not directly, although nothing prevents us from using known linear combinations of separation measures), it is appropriate to suggest a way of handling individual tolerance variability. The issue will be resolved by grouping individual travelers in a certain way. Recall how commuters and opportunities in intermediate destinations have been partitioned. In a manner similar to clustering origin, intermediate and final destinations, so that a single representative separation profile c_{wk} can measure the spatial separation between w and k, we may choose to attribute to each individual commuter $\alpha \in A_w$ a w-specific *threshold* vector $t_w = \{(t_w^{(q)} : q \in Q) \in R^Q\}$. We make now the assumption that each component $t_w^{(q)}$ represents individual α's current maximum tolerable level for separation attribute q. For example, $t_w^{(1)}$ is taken to represent the maximum tolerable travel time $t_w^{(2)}$, the maximum tolerable travel cost, and so forth.

We assume, further, that there exist no structural dependencies between different measures of separation components. This assumption is critical for separation measures such as travel time and travel cost, which tend to be highly correlated, and yet which are behaviorally relevant in their own right. Whenever such dependencies exist, we will implicitly assume that they have been eliminated by a prior reduction to a smaller set of measures. This requirement relates to the sufficiency in the theorems which will be discussed later.

We designate now each situation, in which a traveler α with tolerance levels t_w considers a possible stop for an opportunity β, as a *potential interaction situation* for single-stop work trip chains represented by the triple (α, β, t_w). Given any such potential interaction situation (α, β, t_w), with $\alpha \in A_w$, $\beta \in B_k$ and $t_w \in R^Q$, if the prevailing levels of separation between w and k are given by separation profile

$c_{wk} = (c_{wk}^q : q \in Q)$, then a stop at zone k is realized if and only if the prevailing separation levels between zones w and k do not exceed α's current tolerance levels. In other words, a commuter α stops at an intervening opportunity β if and only if $c_{wk} \leq t_w$, or equivalently, if and only if $c_{wk}^q \leq t_w^q$ for each component $q \in Q$.

It is interesting to note that potential interaction situations lead us to a point of departure from relevant discussions in the framework of disaggregate models. Recall that all we have assumed is that we have only partial information about individual behavior. If, for example, all a commuter needs on the return trip home is a light meal, then we will not assume that this particular commuter knows the optimal (nearest or cheapest) location to buy it. We only assume that the choice will be made for a location that meets the commuter's tolerance criteria as represented by his/her individual threshold vector of spatial separation. It is worth noting that relevant threshold considerations for other factors characterized by accessibility and emissivity profiles are given in Metaxatos (1995). Thus, from a behavioral point of view we are pursuing a satisficing rather than a maximizing theory of individual behavior.

Our objective here is to develop a probabilistic model of potential interaction situations. Models of potential interaction situations are not new. Tellier and Sankoff (1975) and Luoma and Palomäki (1983) have studied similar models for single threshold variables. Smith (1985, 1987a) and Sen and Smith (1995) generalized their approach by developing a *threshold theory of spatial interaction* and embedded it into the more general framework of gravity models of spatial interaction behavior. The basic framework of the threshold theory is extended here to accommodate trip chains occurring between home and work. In particular, we seek to provide working hypotheses and suggest a class of models which will yield meaningful estimates of interaction probabilities $p_c(wk)$, between zonal pairs w and zones k, under any possible separation configuration $c \in C$.

In order to develop such a probabilistic model, we need to define first the relevant outcome (sample) space of possible interaction patterns between mandatory activities in $w \in W$ and discretionary activities in $k \in K$ which may occur during some relevant time period. Recall from the discussion in the previous section how mandatory and discretionary activities are seen in the context of *r-sequential trip-chaining behavior.* Each probability distribution on this outcome space will then constitute a possible probability model of interaction behavior in the given context. The specific probability models of interest for our purposes are designated as *threshold interaction processes* for single-stop work trip chains.

4.1 Threshold Interaction Processes for Work Trip Chains

Since each potential interaction situation for a single-stop work trip chain is described by a triple $\{(\alpha, \beta, t_w) \in A \times B \times R^Q\}$, the relevant individual interaction space for this type of trip chains is given by $\Omega_1 = A \times B \times R^Q$. Hence, Ω_1 denotes the universe of possible *individual potential interaction events* for single-stop work trip chains. To model collections of such events we need to consider the n-fold product set of Ω_1, denoted as $\Omega_n = (\Omega_1)^n$, where n is the universe of commuters stopping in a single intermediate destination. Then each possible occurrence of n individual potential interaction events in Ω_1 may be represented by an element, $\omega = (\omega_r : r = 1, \ldots, n) \in \Omega_n = (\Omega_1)^n$, and designated as a *potential interaction pattern* of size n. If we allow the *null* potential interaction event o (representing the possibility of not stopping at an intermediate destination) to be an element of the corresponding outcome (sample) space Ω then this space is given by $\Omega = \cup_{n \geq 0} \Omega_n$, for all interaction patterns of size n which may occur.

In other words, ω represents a list of n individual single-stop trip chains from which the identity of the individuals involved has been removed, during a given time period. Each one of these commuters is assumed to search among a set of intermediate destinations which meet his/her threshold criteria. However, in this case no notion of a particular type of search process is implied for any commuter because, as we will see later trip-chain frequencies are still characterized by (threshold) gravity models whatever search process may be implied. Thus, we assume that commuters do not exhibit optimizing but rather satisficing behavior, in the absence of relevant information not captured entirely by attitudinal variables of spatial separation.

Assuming that the appropriate partitions of commuters and intermediate destinations has accomplished a much desired within-group homogeneity, the attributes of zones $w \in W$ (involved in the trip between home and work) and the attributes of zones $k \in K$ (where the intermediate discretionary activities occur) will convey all the locational information typically required for analysis. It is convenient to designate the joint realization of these attributes as the *spatial interaction pattern* $s = s(\omega) = \{(w_r, k_r) : r = 1, \ldots, n\}$ associated with each interaction pattern, $\omega = (\omega_r : r = 1, \ldots, n)$. If we allow $S_n = (W \times K)^n$ to denote the set of all spatial interaction patterns of size n, then the relevant outcome space for possible spatial interaction patterns arising from potential interaction patterns in Ω is $S = \cup_{n \geq 0} S_n$, where S_0 denotes the *null* spatial interaction pattern. Therefore, if ω is seen to correspond to a hypothetical trip-diary of n persons, in which each record conveys the information of the particular intermediate activity for exactly one commuter along with individual tolerance levels, then the corresponding $s(\omega)$ records only the location (zonal) information for that particular activity as well as the locations for home and work for those n commuters.

Given the above interaction framework we can describe probabilistically the threshold interaction behavior of commuters in terms of a probability measure on Ω. Each such measure P assigns probabilities $P(G)$ to certain subsets G of Ω, which denote the probability that a realized outcome in Ω will belong to G. The subsets G for which $P(G)$ is defined are called *measurable events* in Ω. Thus, in our context, we require that the subset of potential interaction patterns $\Omega(s) = \{\omega \in \Omega : s(\omega) = s\}$ corresponding to each spatial interaction pattern $s \in S$ be a measurable event in Ω. Hence, if we now write $P(s) = P[\Omega(s)]$, $\forall s \in S$, then each probability measure P on S generates a unique probability function on S (i.e. a nonnegative function P on S satisfying the condition that $\sum_{s\in S} P(s) = 1$).

Since all relevant relations between commuters in W and intermediate activities in K are represented by a given separation configuration $c = (c_{wk} : wk \in W \times K) \in C$, it is appropriate to make any probability measure on S configuration specific, that is, P_c. Then a probability model of spatial interaction behavior consists of a family of probability functions $\{P_c : c \in C\}$ on S which describes interaction behavior between W and K under each possible separation configuration $c \in C$. We designate then the collection of all probability measures $\boldsymbol{P}= (P_c : c \in C)$, on Ω as a *threshold interaction process* (TI-process) for single-stop work trip chains, if and only if $\boldsymbol{P}$ satisfies three regularity conditions of *positivity, symmetry and continuity* in a similar manner as in Sen and Smith (1995).

4.1.1 Frequency Positivity Regularity Condition

The outcome space for possible spatial interaction patterns S contains by definition all finite spatial interaction patterns, and hence includes patterns which are much too large to be meaningful. In this case of single-stop trip chains, there is some upper bound on the size of possible interaction patterns beyond which no pattern can have positive probability. If, for example, each commuter makes a stop between home and work in a given day, then the trip-chain frequencies in either direction cannot be higher than the number of commuters. Thus, no more trip chaining can be observed for that particular period. This, however, is a highly unlikely situation since there may be situations of direct trips from home to work (or vice versa), or even no trips at all. Therefore, since spatial patterns approaching these upper bounds are very unlikely to occur, they are of little relevance here. Hence, we will treat each possible spatial interaction pattern as having positive probability, and "impossible situations" as "extremely unlikely" events. Thus the first regularity condition is:

C1. Positivity: *For all separation configurations $c \in C$ and all spatial interaction patterns $s \in S$ the pattern probability $P_c(s)$ is positive.*

Therefore, even the (null) event of no stops at all at any intermediate destinations for all commuters, who are not habitually non-stoppers, has a small positive probability. This regularity condition can be formalized in terms of wk-frequency variates N_{wk}. In particular, for any $c \in C$, a given zonal wk-pair involved in a single-stop trip chain can occur if and only if the event $\{\omega \in \Omega : N_{wk} \neq 0\}$ has positive probability under configuration c.

4.1.2 Symmetry Regularity Condition

To motivate the second condition we consider all single-stop trip chains during a time period. If we are interested in studying how individual trip chains evolve over time, or in modeling individual trip chains occurring by means of different modes, then the ordering of individual interactions (from destination to destination) is of particular importance. Here, however, we are interested in overall trip-chaining activity patterns which can be represented by mean interaction frequencies. In addition, we believe that it is possible to construct proper probability spaces in such a way that individual interactions are meant to be individual *chained* interactions. In this manner, "exchangeability" on the n individual interaction events defining any realized n-interaction pattern (see Kingman, 1978), or "relabeling" of every n-interaction pattern $\omega = (\omega_1, \ldots, \omega_n) \in \Omega_n$ by means of permutations of the integers $r = 1, \ldots, n$ (see Sen and Smith, 1995) clearly would not affect the probability of occurrence of such chained events. However, we will not pursue such a technical development here.

In this light, given an interaction pattern $s = s(\omega) = [(w_r, k_r) : r = 1, \ldots, n)]$, the labels $r = 1, \ldots, n$ serve only as a means of enumerating the list of interactions. Hence for this type of behavior, the pattern probabilities $P_c(s)$ are completely independent of the order in which the individual interactions in s are labeled. We may impose now the next *symmetry condition* on pattern probabilities:

C2. Symmetry: *For any given separation configuration $c \in C$ and pair of spatial interaction patterns $s = (w_r k_r : r = 1, \ldots, n)$ and s', if s and s' differ only by the ordering of their individual interactions, then $P_c(s) = P_c(s')$.*

As a result, the only behavioral information relevant for our analysis which is contained in each realized spatial interaction pattern s consists of the frequencies of individual interactions $wk \in W \times K$.

An important result following from the symmetry condition is that all patterns with identical interaction frequencies must be equiprobable that is, for all $s, s' \in S$,

$$[N_{wk}(s) = N_{wk}(s') : wk \in W \times K] \Rightarrow P_c(s) = P_c(s') \tag{1}$$

Thus the fundamental property of symmetric processes implies that all probabilistic information about interaction patterns is expressible in terms of their associated (wk)-interaction frequency probabilities. More generally, each spatial interaction process $\boldsymbol{P}= (P_c : c \in C)$ is uniquely expressible in terms of an associated frequency process $\boldsymbol{N}= \{N^c : c \in C\}$ (see Sen and Smith, 1995, Chapter 3).

4.1.3 Continuity Regularity Condition

The final assumption about the pattern probabilities $P_c(s)$ is that small changes in separation values result only in small changes in pattern probabilities. More formally, we postulate that pattern probabilities satisfy the following *continuity condition*:

C3. Continuity: *For any given interaction pattern $s \in S$ separation configuration $c \in C$ and scalar $\epsilon > 0$, there exists a $\delta > 0$ with $|c - c'| < \delta$ sufficiently small to ensure that $|P_c(s) - P_{c'}(s)| < \epsilon$ holds for all configurations $c' \in C$.*

In other words, we require that $P_c(s)$ be a continuous function of the parameter vector c. This assumption may be inappropriate at the level of the individual commuter participating in a trip chain. For example, a small rise, in bridge tolls, or the introduction of congestion tolls (i.e. a small change in the configuration $c \in C$), produces quite possibly radical changes in individual trip-chaining behavior (for a relevant discussion see Hirschman *et al*, 1995; Williams, 1995). At the aggregate level, however, such individual effects tend to be insignificant. Even when all individuals exhibit threshold behavior, their threshold levels generally differ, yielding thus a smoother pattern in the aggregate.

A fourth assumption, that of *Threshold Positivity*, $P_c^n(T_r \geq t_r) > 0$, can be imposed to ensure that realized wk-interactions are always possible. This condition, however, is not essential from a behavioral viewpoint, but rather serves to facilitate formal developments by avoiding the need for special analysis of degenerate cases (see Sen and Smith, 1995).

4.2 Independent Threshold Interaction Processes for Work Trip Chains

Having discussed the requirements for general threshold interaction processes we develop in this section, conditions for *independent* threshold interaction processes for single-stop work trip chains. In particular, we will give three axioms which serve to provide a set of null hypotheses for the investigation of trip-chaining behavior. In cases where these hypotheses cannot be rejected on the basis of subsequent observations, such interaction behavior may be described by the gravity-type family of models.

In fact, because of the *Poisson Characterization Theorem* (see Smith, 1987a, 1987b; Sen and Smith, 1995) this type of behavior is characterized by such type of models. Moreover, under those hypotheses, the interaction probabilities can be estimated by standard maximum likelihood techniques.

We will give now statistical independence assumptions about certain relevant attributes of potential interaction patterns. For each potential interaction pattern $\omega = [(\omega_r = \alpha_r, \beta_r, t_{wr}) : r = 1, \ldots, n]$, these relevant attributes can be represented by three sets of random variables. The first set $(W_r,\ r = 1, \ldots, n)$ denotes the origin and final destination of commuter α_r. The second set $(K_r,\ r = 1, \ldots, n)$ denotes the location of the discretionary intermediate activity β_r. Finally, the third set $(T_{wr},\ r = 1, \ldots, n)$ denotes the threshold levels for each separation measure c^q_{wk} at each level $q \in Q$, of commuter α_r in a realized pattern of size n. Interestingly, by treating threshold levels as random variables we may observe another point of departure from disaggregate models. In particular, if threshold levels are seen as negative utility, then not only do we claim that this utility is not specific to each individual commuter, but also that it is not, even partially, known. In a typical random-utility environment the information for the estimation of the systematic part of the individual utility would have to come from a survey treated as a random sample.

Define now the joint distribution of the random variables $T^n_w = (T_{wr} : r = 1, \ldots, n)$ on Ω_n, for all possible threshold patterns of size n, $t^n_w = (t_{wr} : r = 1, \ldots, n)$ by

$$\begin{aligned} P^n_c(T^n_w \geq t^n_w) &= P^n_c(T_{wr} \geq t_{wr},\ r = 1, \ldots, n) \\ &= P^n_c\{[\omega \in \Omega_n : T_{wr}(\omega) \geq t_{wr}(\omega)],\ r = 1, \ldots, n\} \end{aligned} \tag{2}$$

Then for each spatial pattern $s \in S_n$ and threshold pattern $t^n_w = (t_{wr},\ r = 1, \ldots, n)$, the probable occurrence of a threshold pattern at least as large as t^n, given that the spatial pattern s occurs, is

$$\begin{aligned} P^n_c(T^n_w \geq t^n_w|s) &= \frac{P^n_c[s, (T^n_w \geq t^n_w)]}{P^n_c(s)} \\ &= \frac{P^n_c\{\omega \in \Omega_n : s(\omega) = s, T_{wr}(\omega) \geq t_{wr}(\omega), r = 1, \ldots, n\}}{P^n_c\{\omega \in \Omega_n : s(\omega) = s\}} \end{aligned} \tag{3}$$

Finally, if for each potential interaction pattern $\omega \in \Omega$ we denote the corresponding frequency profile of potential interactions by $n = n(\omega) = (n_{wk} : wk \in W \times K)$, then a threshold interaction process $\boldsymbol{P} = \{P_c : c \in C\}$ is designated as an independent TI-process if and only if $\boldsymbol{P}$ satisfies the three axioms of *locational, frequency and threshold independence* discussed next.

4.2.1 A1. Axiom of Locational Independence

To motivate the first axiom, we observe that it is highly unlikely that intermediate destinations are extremely concentrated in space due to congestion effects. For example, for any reasonable number of trips it is virtually impossible for all trips to stop at the same location. On the other hand, assume that there may be a case where commuters live close to one another, work close to one another, and leave their cars at the same parking lot to take rapid transit to work; or, consider the case of friends who like to shop together and happen to live and work close to one another. In both cases the occurrence of one trip chain, from the residential zone to the intermediate destination zone to the employment zone, may imply the occurrence of several trips going through the same zones. More generally, there may exist a variety of *contagion effects* or *band wagon effects* which can lead to identical interaction choices by many individuals.

While individual interdependencies over space are important from a research point of view, our objective remains to model the overall effect of spatial separation of the discretionary activities on interaction behavior. Hence, we hypothesize that the influence of particular types of interdependencies among commuters is minimal, and thus individual interaction decisions can be treated as statistically independent events. More formally, for all non-null spatial potential-interaction patterns $s = (w_r k_r : r = 1, \ldots, n) \in S$ and separation configurations $c \in C$,

$$P_c^n(s) = P_c^n(w_1, \ldots, w_n, k_1, \ldots, k_n) = \prod_{r=1}^{n} P_c^n(W_r = w_r, K_r = k_r) \tag{4}$$

where $P_c^n(w_1, \ldots, w_n, k_1, \ldots, k_n)$ denotes the joint distribution of locational realizations w_r and k_r, $r = 1, \ldots, n$. In other words, for any given interaction (w_r, k_r) in an interaction pattern of size n, we assume that no properties of the other realized interactions $(w_\sigma, k_\sigma : \sigma \neq r)$ influence the likelihood that (w_r, k_r) will occur. It is assumed implicitly that populations engaged in a single-stop trip chain between home and work are sufficiently large to minimize the influence of any individual opportunities for discretionary activity.

Violation of this assumption can happen, for example, if the total trip frequencies present so little variation that can be described by a deterministic rather than a Poisson-based model (even if certain individual intermediate activities remain the same or show little variation from day to day). This may be more relevant for the home-to-work trip chain where time constraints, for example, may cause much more regularity in the observed travel patterns. Conversely, trip chain factors other than time may cause an eclectic behavior with respect to stopping at particular intermediate destinations. Thus we may, alternatively, need to consider mixtures of distributions and build, respectively, models of *restricted*, in the first place, or *extra variation*, in the second case. Although that level of detail is not pursued here, we believe that these important issues are better resolved on a case by case basis. Exploratory data analysis may reveal dominance of one or the other type of behavior and imply a particular type of modeling needs.

4.2.2 A2. Axiom of Frequency Independence

In addition to the first hypothesis, we may now assume that wk-frequencies are statistically independent. In other words, if the number of wk-trips are high, this need not imply that the number of wk'-trips is low. Therefore, it is implicitly assumed that the population of commuters A_w and opportunities in intermediate destinations B_k are sufficiently large to allow the obvious frequency dependencies for any given individual commuter to be ignored. For example, if an individual α regularly visits the same supermarket θ on the way back to home, then it may be inferred that in general, α does not frequent other supermarkets. However, if shopping trips for commuters living in zone i and working in zone j to zone k (location of the supermarket) are high, this need not imply that shopping trips for the same commuters to another supermarket in a different zone k' are low.

More formally, for all potential-interaction frequency profiles $\boldsymbol{n}= \{(n_{wk} : wk \in W \times K) \in Z_{+}^{W\times K}\}$ and separation configurations $c \in C$,

$$P_c(\boldsymbol{n}) = P_c(N_{wk} = n_{wk} : wk \in W \times K) = \prod_{w\in W}\prod_{k\in K} P_c(N_{wk} = n_{wk}) \tag{5}$$

where,
$P_c(\boldsymbol{n})$ – the joint distribution of potential trip frequencies $N_{wk} = n_{wk},\ wk \in W \times K$.
$P_c(N_{wk} = n_{wk})$ – the associated marginal distributions.

Violation of this assumption can happen, for example, if there is reason to believe that each possible factor that can be hypothesized to affect the level of the overall trip-chaining behavior (such as, travel distance, travel time, flexible work schedule, family status, full time employment, car availability, carpooling, individual tastes and preferences, familiarity with the area, stress, overall economic growth, technological change, etc.), will change from year to year. In other words, if the sizes of the populations of commuters and opportunities for discretionary activity are influenced by factors which are not consistent with Poisson randomness, there is need to consider more specific types of non-Poisson models.

4.2.3 A3. Axiom of Threshold Independence

Finally, it is hypothesized that the attitudes toward trip-chaining spatial interaction wk are intrinsic to individual travelers and are influenced neither by locational factors nor by the presence or absence of other individuals. This axiom asserts that threshold values of commuters are independent of the number of other commuters living and working at the same zones.

Again, it is easy to imagine a number of situations (e.g. among family members) where this type of independence would be questionable. It is assumed, however, that the number of commuters participating in trip chains is large enough so that individual interdependencies can be ignored. Hence, within any given separation configuration $c \in C$, all effects of space on trip-chaining interaction behavior are assumed to be captured by attitudinal threshold variables alone. More formally, for all threshold patterns t_w^n, spatial interaction patterns $s \in S_n$ and separation configurations $c \in C$,

$$\begin{aligned} P_c^n(T_w^n \geq t_w^n | s) &= \\ P_c^n(T_{w1} \geq t_{w1}, \ldots, T_{wn} \geq t_{wn} \mid & \\ W_1 = w_1, \ldots, W_n = w_n,\ K_1 = k_1, \ldots, K_n = k_n) &= \\ \prod_{r=1}^{n} P_c^1(T_{w1} \geq t_{wr}) & \end{aligned} \tag{6}$$

where,
T_{wr} – the threshold levels for the r-th commuter in a realized pattern of size n.
$P_c^n(T_{w1} \geq t_{w1}, \ldots, T_{wn} \geq t_{wn} | w_1, \ldots, w_n,\ k_1, \ldots, k_n)$ – the conditional distribution of threshold realizations $T_{wr} \geq t_{wr},\ r = 1, \ldots, n,$ given the locational realizations $W_r = w_r,\ K_r = k_r,\ r = 1, \ldots, n$.
$P_c^1(T_{w1} \geq t_{wr}),\ r = 1, \ldots, n$ – the associated marginal distributions.

4.3 Realized Threshold Frequencies for Work Trip Chains

The main purpose of axioms A1, A2, and A3 of locational, frequency and threshold independence, respectively, is to provide a set of working null hypotheses. These hypotheses are not directly testable since a potential interaction situation (α, β, t_w) is not in general observable. A potential interaction situation may result, however, in a *realized interaction*, namely an interaction in which the separation profile $c_{wk} = \{c_{wk}^q : q \in Q\}$ between commuter $\alpha \in A_w$ and opportunity $\beta \in B_k$ does not exceed α's threshold profile t_w in any component. Realized interactions may then provide indirect tests for these hypotheses.

This realized-interaction property for each pair $(w, k) \in W \times K$ separation configuration $c \in C$, and each r-th component $\omega_r = (\alpha_r, \beta_r, t_{wr})$ of potential interaction patterns $\omega \in \Omega$ can be represented by a zero-one variable, $\delta_{wk}^{cr}(\omega)$, of the form:

$$\delta_{wk}^{cr}(\omega) = \begin{cases} 1, & \text{if } \alpha_r \in A_w,\ \beta_r \in B_k, \text{ and } T_{wr} \geq c_{wk} \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

Then, for each pair $(w, k) \in W \times K$, the number $N_{wk}(\omega)$ of realized interactions between w and k in pattern ω under configuration c is given by:

$$N_{wk}^{c}(\omega) = \begin{cases} 0, & \omega \in \Omega_0 \\ \sum_{r=1}^{n} \delta_{wk}^{cr}(\omega), & \omega \in \Omega_n, n > 0 \end{cases} \tag{8}$$

The random variables $N_{wk}^{c}(\omega)$ are designated as the *threshold frequencies* of realized spatial interactions between commuters in origin-final destination zonal pairs w and opportunities for discretionary activity at locations k under configuration c. The probability distribution of these integer-valued random variables $N_{wk}[s(\omega)]$ under each probability measure P_c in an independent TI-process $\boldsymbol{P}$ is then defined by,

$$P_c(N_{wk} = n_{wk}) = \sum_{s \in S(n_{wk})} P_c(s) \tag{9}$$

The associated *mean realized-interaction frequency* is given for all $wk \in W \times K$ and $c \in C$ by,

$$E(N_{wk}^{c}) = \sum_{s \in S} N_{wk} P_c(s) = \sum_{n_{wk}} n_{wk} P_c(n_{wk}) \tag{10}$$

Moreover, from the *Threshold-Frequency Theorem* (Smith, 1987a, Section 2.2; Sen and Smith, 1995, Theorem 3.4, Section 3.7.3), we obtain that the observable interaction frequencies $(N_{wk}^{c} : wk \in W \times K)$ are Poisson distributed; that is,

$$P_c(N_{wk} = n_{wk} : wk \in W \times K) = \prod_{w \in W} \prod_{k \in K} \frac{E(N_{wk}^{c})^{n_{wk}}}{n_{wk}!} \exp[-E(N_{wk}^{c})] \tag{11}$$

with mean realized-interaction frequencies $E(N_{wk}^{c})$ in $\boldsymbol{P}$, given for all $wk \in W \times K$ and $c \in C$ by,

$$E(N_{wk}^{c}) = E_c(N) P_1^c(W_1 = w, K_1 = k) P_1^c(T_{w1} \geq c_{wk}) \tag{12}$$

where,

$P_1^c(W_1 = w, K_1 = k)$ – the marginal probability of a single potential-interaction situation for a traveler commuting between the home-work zonal pair w and stopping at zone k;

$P_1^c(T_{w1} \geq c_{wk})$ – the marginal probability of a single potential-interaction situation for a commuter having threshold levels at least as large as c_{wk};

$E_c(N)$ – the expectation of the random variable $N = \sum_{wk} N_{wk}^{c}$ denoting the total frequency of potential interactions.

It can be proved (Sen and Smith, 1995, Theorem 3.3, Section 3.7.2) that for any independent TI-process $\boldsymbol{P} = \{P_c : c \in C\}$ and separation configuration $c \in C$,

$$E_c(N_{wk}) = E_c(N) P_1^c(W_1 = w, K_1 = k) \tag{13}$$

Hence, (12) gives,

$$E_c(N^c_{wk}) = E_c(N_{wk})P^c_1(T_{w1} \geq c_{wk}) \tag{14}$$

Therefore, for any potential stops at intermediate destinations, larger component values of the separation profile c_{wk} will reduce the probability that travelers' threshold levels will exceed these values, and hence reduce the probability of a realized stop. In other words, threshold behavior is seen to reduce the mean frequency of potential intermediate stops to a smaller mean frequency of realized stops at intermediate destinations. Thus, the threshold trip-chaining behavior as presented above may be seen to act as a filter ("thinning") operator on the Poisson process $(N_{wk} : wk \in W \times K)$.

In this context, the formal role of the threshold independence axiom, together with the symmetry regularity condition, is to ensure that the filter operator is spatially homogeneous over the $W \times K$ space. In this light, the *Threshold-Frequency Theorem* can be viewed as a special instance of the more general result that every homogeneous filtering of a Poisson process yields a new Poisson process with reduced mean intensity measure (see Matthes *et al*, 1978, Proposition 1.13.7).

From (12) we obtain for all $wk \in W \times K$ and $c \in C$,

$$E(N^c_{wk}) = E_c(N)P^c_1(W_1 = w)P^c_1(K_1 = k|W_1 = w)P^c_1(T_{w1} \geq c_{wk}) \tag{15}$$

We now let the functions $A_c : W \rightarrow R_{++}$ (an origin-final destination weight function under configuration c), $B_c : K \rightarrow R_{++}$ (an intermediate destination weight function under c), and $F_c : R^{Q_c} \rightarrow R_{++}$ (a separation-friction function between w and k under c) be defined, respectively, by,

$$\begin{aligned} A_c(w) &= E_c(N)P^c_1(W_1 = w) \\ B_c(k) &= P^c_1(K_1 = k|W_1 = w) \\ F_c(c_{wk}) &= P^c_1(T_{w1} \geq c_{wk}) \end{aligned} \tag{16}$$

Thus, the origin-final destination weight function $A_c(w)$ is precisely the expected number of potential-interaction situations involving individual commuters between an origin-final destination zonal pair w. The intermediate destination weight function $B_c(k)$ is precisely the conditional probability that any given potential interaction situation will involve an opportunity for an (intermediate) discretionary activity at zone k, given a commuting trip in w. Finally, the separation-friction function $F_c(c_{wk})$ is the threshold distribution summarizing the behavioral attitudes of the population toward various types of spatial separation costs and other measures of spatial separation between w and k.

The threshold theory discussed above does not specify those functions exactly. It suggests, however, both the types of variables and functional relationships which should play an important role in such specifications. For example, the above interpretations serve to support common specifications of the origin and destination functions as increasing functions of the sizes of populations A_w and B_k, respectively, and the specification of the separation function as a decreasing function of all its arguments. For the separation function, in particular, it can be proved (Sen and Smith, 1995, Chapter 2) that the multivariate exponential deterrence function, as defined first by Sen and Sööt (1981), is general enough for most applications of interest. Thus, given the definitions in (16), we obtain from expression (15),

$$E(N^c_{wk}) = A_c(w)B_c(k)F_c(c_{wk}) = A_c(w)B_c(k)\exp(-\sum_{q\in Q}\theta_q c^q_{wk}) \tag{17}$$

where $\boldsymbol{\theta} = \{(\theta_q : q \in Q) \in R^Q_+\}$ is a cost sensitivity vector of dimensional parameters.

Therefore, the mean realized-interaction frequencies $E(N^c_{wk})$ are representable by a gravity model, which is a monotone gravity model if we require that F_c is nonincreasing. Obviously, the exponential function in (17) satisfies this requirement. In fact, it is proved (Proposition 4.2 of Section 4.3.2, Sen and Smith, 1995) that each independent TI-process $\boldsymbol{P}$ is *mean-representable* by a monotone gravity model if and only if these mean realized-interaction frequencies $E(N^c_{wk})$ are representable by that gravity model.

An additional formal consequence of the Threshold-Frequency Theorem is that the threshold frequencies $N_{wk} : wk \in W \times K$ are conditionally multinomially distributed for each given level of the total frequency of realized interactions $N = \sum_{wk} N_{wk}$ under configuration c (Smith, 1986, Corollary 3.1). In particular, if for each configuration c, we now employ (17) to define the associated interaction probability distribution p_c over wk-pairs by,

$$p_c(wk) = \frac{A(w)B(k)F(c_{wk})}{\sum_{wk} A(w)B(k)F(c_{wk})} \quad \forall wk \in W \times K \tag{18}$$

then as a direct consequence of the Threshold-Frequency Theorem it follows that

$$P(N^c_{wk} = n_{wk} : wk \in W \times K | N^c = n) = n! \prod_{w\in W}\prod_{k\in K} \frac{p_c(wk)^{n_{wk}}}{n_{wk}!} \tag{19}$$

Hence, for any given level of observed interaction frequencies n the realized frequencies can be treated as n independent random samples from the interaction probability distribution p_c. Moreover, the relative realized frequencies $(n_{wk}/n, wk \in W \times K)$ are the maximum likelihood estimators of $p_c(wk)$, and thus constitute sufficient statistics for estimation and testing.

4.4 Extending the Theoretical Framework

The threshold theory as described above views the trip-chain flows and the corresponding flow-frequencies as an independent Poisson process with mean intensity measure representable by an exponential (separable) gravity model. In this model the separation-friction function $F_c(c_{wk})$ is the threshold distribution summarizing the behavioral attitudes of the population toward various types of spatial separation costs and other measures of spatial separation between w and k. Although the multivariate exponential deterrence function is a general enough specification for F_c, no other specifications for the origin- and the destination-weight functions have been offered explicitly.

Spatial separation is arguably a very important factor toward spatial interaction. There are cases, however, in which neighboring intermediate destinations attract different number of commuting trips. It may be hypothesized, therefore, that in those cases a host of factors related to each intermediate destination affects the attitudes of the population toward trip chaining. Certain intermediate destinations, for example, may be viewed by the prospective commuter as more "accessible", or more "attractive' than other ones. Familiarity with the area, store size and closing time, product variety, price levels, agglomerations of opportunities, etc. are some of the factors that may affect this perception. It is thus desirable to extend the threshold theory so that these attitudes are also considered. In addition, we may further extend the discussion so that factors relevant to the origin location affecting individual attitudes toward trip chaining (such as car availability, family size and income, gender and family status, and age and physical capacity of the commuter) are also considered.

Theoretical developments to accommodate these additional considerations are pursued in Metaxatos (1995). Metaxatos shows that if the intermediate-destination weight function, in a context similar to (16), is defined precisely as the joint probability of the event that any given potential interaction situation will involve an opportunity for an (intermediate) discretionary activity at zone k, given a commuting trip in w, and the event that threshold levels (behavioral attitudes) of the population toward various types of accessibility considerations with respect to stopping at an intermediate destination k have not been exceeded, then the mean realized-interaction frequencies are still representable by a gravity model in which measures of accessibility costs to intermediate destinations k, and separation costs between w and k are explicitly taken into account. Moreover, separation-accessibility threshold behavior is seen to reduce the mean frequency of potential intermediate stops to an even smaller mean frequency of realized stops at intermediate destinations than the one realized under separation-threshold behavior only.

If, further, the origin-final destination weight function is defined as the expected number of potential-interaction situations involving individual commuters between an origin-final destination zonal pair w, who do not exceed their emissivity thresholds, then the mean realized-interaction frequencies are still representable by a gravity model, in which measures of emissivity of origins i, accessibility costs to intermediate destinations k, and separation costs between w and k are explicitly taken into account (see Metaxatos, 1995). In addition, separation-emissivity-accessibility threshold behavior is seen to reduce the mean frequency of potential intermediate stops to an even smaller mean frequency of realized stops at intermediate destinations than the one realized under separation-accessibility threshold behavior only.

4.5 Behavioral Implications for the Theoretical Framework

The behavioral implications of the *threshold theory* for single-stop trip chains, as presented above, are best understood by focusing on the linkages between the theory and other aggregate and disaggregate approaches. We discuss, first, the distributional assumptions embedded in macro and micro models and compare them with the assumptions postulated by the threshold theory. Then we compare the total and conditional frequencies obtained by entropy-maximizing and disaggregate random-utility models against frequencies from the threshold theory, and discuss the relevant implications.

4.5.1 Distributional Assumptions

Current aggregate models involving the *maximum-entropy principle* (Wilson, 1967, 1970) offer no discernible notion of individual choice behavior. Even the more behaviorally oriented *cost-efficiency principle* (Smith, 1978) is formulated solely in terms of macro behavioral axioms. On the other hand, current disaggregate models establish only a weak link to observable macro behavior. In particular, random-utility models postulate an explicit utility-maximizing model of individual choice behavior, but offer no clear principles of aggregation. Thus all observable macro behavior is represented by a population distribution of unobserved utility components, which is predetermined. Different choices for this distribution, such as the extreme-value distributions in multinomial logit models (McFadden, 1974), or the normal distributions in multinomial probit models (Hausman and Wise, 1978; Daganzo, 1979) can lead to different results (Horowitz, 1981).

Smith (1987a) points to research suggesting the specific types of behavioral contexts for which these distributional assumptions might be most appropriate (see, for example, Kagan *et al*, 1973, for normal distributions; McFadden, 1974, pp. 105-142, for extreme-value distributions; Strauss, 1979, and Smith, 1984, for generalized extreme-value distributions). More general findings, according to Smith (1987a), attempt to justify certain distributional types as limiting forms for large families of distributions (see, the Lindeberg-Feller Theorem, Theorem 7.2.1, in Chung, 1968; and the Fisher-Tippett-Gnedenko Theorem, Theorem 1.6.2, in Leadbetter *et al*, 1983). Of course, all the above results are asymptotic and tell us little about small samples.

The threshold theory, however, yields exact distributions on the basis of certain independent assumptions, and thus may offer a link between macro and micro approaches. This is because the theory postulates at the micro level, as first envisioned by Smith (1987a), a simple satisficing individual choice model which focuses on the individual's decision on whether or not to pursue an intermediate activity under consideration, by stopping at an intermediate destination between home and work. Moreover, at the macro level, the theory postulates a simple population model in which potential trip-chaining situations are assumed to exhibit certain independence assumptions over space. In addition, given the particular specification of the separation-friction function $F_c(c_{wk}) = \exp(-\sum_{q\in Q} \theta_q c^q_{wk})$ which is general enough for most applications of interest, that of the origin-weight function $A_c(w)$, which may be seen to summarize measures of emissivity of origin zones (Metaxatos, 1995), and the intermediate-destination weight function $B_c(k)$, which may be seen to account for measurable attributes of intermediate destinations (Metaxatos, 1995), the threshold theory yields an explicit model of mean trip-chain frequencies, which is similar to the exponential models derived from entropy maximization, and much more explicit than disaggregate random-utility models. Thus, the threshold theory may be viewed as suggesting the possibility of developing a spectrum of aggregate-disaggregate models for different types of trip-chaining behavior.

4.5.2 Analysis of Conditional and Total Interaction Frequencies

We have seen that the threshold frequencies $N^c_{wk} : wk \in W \times K$, for each configuration c, represent total numbers of individual trip-chain frequencies occurring during some relevant period. In the entropy-maximizing approach these frequencies are prespecified by constraints of the form $\sum_j N_{ij} = \bar{N}_i$ (origin constraints), $\sum_i N_{ij} = \bar{N}_j$ (final destination constraints), $\sum_{ij} N_{ijk} = \bar{N}_k$ (intermediate destination constraints), and $\sum_k N_{ijk} = N_{ij}$ (origin-final destination constraints).

Similarly, in disaggregate random-utility approaches the focus is on the choice behavior of a randomly sampled individual, rather than on how many choices are made. Hence, all information about total frequencies of choices must again be specified exogenously. In particular, one could possibly estimate the expected number of trips N_c by prespecifying the exact number of potential trips n and computing the conditional expectation $E(N_c|N = n)$; or, alternatively, by exogenously specifying an exact distribution for the number of potential trips N and computing the unconditional expectation $E(N_c) = E_N[E(N_c|N)]$.

In contrast to these approaches, the threshold theory yields exact Poisson distributions for frequency totals N^c_{wk} which are entirely determined by their associated mean values $E(N^c_{wk})$. Moreover, if the separation-friction function $F(.)$, the origin-weight function $A(.)$, and the intermediate-destination weight function $B(.)$ can be specified up to some small unknown number of parameters, then by treating the origin and destination functions as finite parameter vectors, one can carry out standard maximum-likelihood estimation and testing. In particular, if functions $F(.)$, $A(.)$ and $B(.)$ have (or can be approximated by) the exponential form, then the resulting estimation procedure would yield exponential (generalized) gravity models which are similar to those derived by the entropy-maximizing framework. Thus, entropy-based gravity models (derived from Wilson's probabilistic framework under Stirling's approximation) can be seen as an *approximation* to the gravity models derived by the threshold theory.

Finally, if we are interested in the relative frequency distribution of intermediate destinations k given commuting trips between zonal pairs w, then the resulting conditional probabilities

$$p_c(k|w) = \frac{p_c(wk)}{p_c(w)} = \frac{A(w)B(k)F(c_{wk})}{\sum_k A(w)B(k)F(c_{wk})} = \frac{B(k)F(c_{wk})}{\sum_k B(k)F(c_{wk})} \quad \forall wk \in W \times K \qquad (20)$$

are formally equivalent to a disaggregate multinomial logit model with deterministic utility component given by $V_{k|w} = \ln[B(k)] + \ln[F(c_{wk})]$, for each specification of $F(.)$. Therefore, given the above definitions of functions $B(.)$ and $F(.)$, this class of disaggregate models obtain a new behavioral interpretation under the threshold theory.

4.6 Limitations of the Theoretical Framework

A number of issues which limit the range of application of the threshold theory are now presented. These issues relate to the assumptions made during the development of the theory and range from variations in the individual threshold levels to the measurement of spatial separation between commuters and intermediate opportunities to the effects of the independence hypotheses.

4.6.1 Tradeoffs Among Separation Measures

The threshold theory postulates a separation-threshold vector $(t_w^1, \ldots, t_w^q)$ of maximum tolerable levels for the corresponding separation measures $(c_{wk}^1, \ldots, c_{wk}^q)$, and implicitly assumes that there are no substitution effects among those measures for each individual traveler. Moreover, the theory assumes that no substitution between separation measures takes place. Of course, nothing prevents us from using known functional relationships among different measures. If, however, individual threshold levels are subject to stochastic variations, these variations are indistinguishable from tradeoff effects in the models developed in (17).

4.6.2 Measurement of Spatial Separation

Another issue relates to the measurement of spatial separation between commuters and opportunities in intermediate destinations. It is assumed that all commuters living and working in the same origin-destination zonal pair w perceive the same separations $(c^1_{wk}, \ldots, c^q_{wk})$, between their commuting and a deviation for an intermediate stop at zone k. This requires that the zones be sufficiently small to preclude within-zone variation of these measures and that values of these measures be sufficiently prominent to preclude significant perceptual variations among individuals. Again, the theory assumes that origins zones are homogeneous and intermediate destinations are prominent enough so that their attributes can be perceived in a similar manner by commuters in w.

These assumptions are particularly strong, especially with subjective measures of separation, such as information uncertainty about distant opportunities. The same problem, of course, is seen in the disaggregate random-utility models where separation measures appear as part of the deterministic component in individual utility functions; it also appears in the aggregate entropy-maximizing models where such measures appear as part of the constraints or the objective function.

4.6.3 The Effects of the Independence Hypotheses

During the discussion of the independence assumptions, a number of cases were presented where these assumptions could not hold. It would not be difficult to construct additional examples, and indeed, each one would require a special treatment. Thus the independence axioms may be viewed as a set of null hypotheses to employ in testing for the presence of spatial dependencies. The fact that these axioms imply an exact distribution theory renders them capable of being rejected on statistical grounds alone. It would be prudent to attempt to reject these hypotheses before attempting to develop more elaborate models. Indeed, Smith (1987a) envisioned a role for the threshold theory as a *benchmark model* against which more complex types of spatial interaction behavior can be compared.

5 Empirical Analysis

The gravity model derived in (17) was estimated using data from a home interview survey. Since we are considering only one separation configuration, the results reported in this chapter remain relevant for that particular configuration. The results from the calibration of the model are very promising toward the development of trip-chaining models which can either stand alone or be incorporated into the more general urban travel demand forecasting models.

The threshold gravity model in (17) has been estimated by an adaptation of the modified scoring procedure (Yun and Sen, 1994), a maximum likelihood procedure. The modified scoring procedure has demonstrated superior performance against other maximum likelihood estimation procedures for much larger problems (see Yun, 1992; Yun and Sen, 1994; Sen and Smith, 1995, Chapter 5). In this particular application the origin-final destination weight function $A_c(w)$, and the intermediate destination weight function $B_c(k)$ are treated as parameters. The modified scoring procedure starts with the DSF procedure (see Sen and Smith, 1995, Chapter 5). The DSF procedure balances a two-dimensional matrix with rows the origin-final destination zonal pairs, and columns the intermediate zones by computing the balancing coefficients A_w's and B_k's. As a result, T_{wk}'s become a function of the vector $\boldsymbol{\theta}$. The modifying scoring procedure then expresses small changes of T_{wk}'s as a function of small changes in $\boldsymbol{\theta}$ and uses the normal equations in the maximum likelihood estimation to introduce iteratively a correction in the current value of $\boldsymbol{\theta}$. Details of the particular adaptation are presented in Metaxatos (1995).

Maximum likelihood estimates of θ's possess very desirable asymptotic properties. In particular such estimates are proved (Sen and Smith, 1995, Chapter 5) to be *efficient, strongly consistent, asymptotically normal* and *robust* for realistic departures from the Poisson assumption. Even for small sample sizes (case of interest here), the same authors report that the parameters estimates may not present noticeable bias or deviation from normality. This guarantees that tests and procedures, such as those for determining sample sizes, based on the normal distribution can be safely carried out.

5.1 Data Preparation and Related Issues

The model derived in (17) was estimated based on data from a home interview survey. In particular, the 1990 Chicago Area Transportation Study (CATS) Household Travel Survey (Ghislandi *et al*, 1994) was used. The survey was a self-administered mail-out mail-back survey which targeted individuals at their homes sampled from electric meter addresses. It collected travel and demographic data for individuals of age 14 and older. The data base includes 19,314 households (24% average response rate) from the six-county region of Northeastern Illinois, or 40,568 persons who made 162,755 trips on a particular day (Thursday).

The survey data were geocoded by CATS staff using the quarter-section coding method. Over 15,000 quarter-sections comprise the entire Chicago metropolitan area. A quarter-section is a zone of approximately 0.25 square miles (almost 0.65 square kilometers) or 160 acres. Trips participating in single-stop work trip chains were next identified. 1,454 commuters from home to work, and 2072 travelers in the reverse commute were found stop once for an intermediate activity.

The quarter section geography proved to be a very detailed system for the data available. We sought thus a more aggregate zone system and adopted the 1992 CATS regional 1640-zone system. This zone system retains the finer grid of the quarter-section system in the more urbanized areas (for example, in the Chicago Central Business District area the two systems coincide) while aggregating the quarter-sections to 9 or 36 square mile-zones in the sparsely-populated areas.

Let us examine the effect of such an aggregation. Assume that a number of zones has been aggregated from i_1 to i_2 and from j_1 to j_2. Then,

$$T_{ij} = A_i B_j F_{ij} \Rightarrow \sum_{i_1}^{i_2} \sum_{j_1}^{j_2} T_{ij} = \sum_{i_1}^{i_2} \sum_{j_1}^{j_2} A_i B_j F_{ij} \tag{21}$$

We may write now,

$$\begin{aligned} T_{kl} &= \sum_{i_1}^{i_2} \sum_{j_1}^{j_2} T_{ij} = \frac{\sum_{i_1}^{i_2} \sum_{j_1}^{j_2} T_{ij}}{\sum_{i_1}^{i_2} \sum_{j_1}^{j_2} A_i B_j} \sum_{i_1}^{i_2} \sum_{j_1}^{j_2} A_i B_j \\ &= \frac{\sum_{i_1}^{i_2} \sum_{j_1}^{j_2} T_{ij}}{\sum_{i_1}^{i_2} \sum_{j_1}^{j_2} A_i B_j} \sum_{i_1}^{i_2} A_i \sum_{j_1}^{j_2} B_j = A_k B_l F_{kl} \end{aligned} \tag{22}$$

where,

$$\begin{aligned} A_k &= \sum_{i_1}^{i_2} A_i \\ B_l &= \sum_{j_1}^{j_2} B_j \\ F_{kl} &= \frac{\sum_{i_1}^{i_2} \sum_{j_1}^{j_2} A_i B_j F_{ij}}{\sum_{i_1}^{i_2} \sum_{j_1}^{j_2} A_i B_j} \end{aligned} \tag{23}$$

Therefore, if we aggregate our zonal system from a set of ij-zones to a set of kl-zones we obtain a new gravity model T_{kl}. The new model has separation-friction function $F(c_{kl})$, a weighted average of the initial separation-friction function $F(c_{ij})$, as in (23). Most importantly, the new model predicts the same total number of trips between aggregated zones as the former model for the more detailed zone system. This result is known as the *compressibility property* of the gravity model, as was indicated first by Bearwood and Kirby (1975). In this application we choose to ignore the possibility that when the aggregated zones are large, the functional form of F may not be the same.

In this application separate trip tables were built in order to investigate the attitude of commuters during the home-to-work (basically in the morning) and work-to-home (basically in the afternoon) commute. In those tables one dimension is always reserved for the home-work zonal pairs and the other dimension for the intermediate destinations. The home-to-work one-stop trip chain is represented by a 51×5 trip table (in Table XI), while the reverse trip chain by a 80×7 trip table (in Table XIII).

In the work-to-home table, two individual characteristic cases were isolated. The first relates to a commuter, who on the way back from work deviated over 40 miles for an intermediate stop. The second case relates to another commuter, who on the way back from work continued past the home location for almost 20 more miles for an intermediate stop. These two cases can hardly be considered typical trip-chaining behavior. Therefore, the results reported for the work-to-home trip chains do not take into account those two cases. As a result, we ended up with 63 observations for the home-to-work single-stop trip chain and 87 observations for the reverse trip chain.

Zone-to-zone airline distances (in miles) were used initially as separation measures because they are easy to compute. A disadvantage of using distance as a separation measure is that it is implicitly assumed that each commuter perceives each mile the same way as every other mile. Thus, we obtained also data for travel times (in minutes) from CATS. Travel times in this case are automobile shortest routes (automobile skims) between zones. They were computed for the 1640-zone system using a user-equilibrium traffic assignment algorithm (see Patriksson, 1994).

5.2 Covariance of θ's

Once the parameters θ have been estimated from the modified scoring procedure, we may compute their covariance matrix and carry out hypotheses testing. The computation of the covariance matrix of Poisson gravity models parameters is given by Sen and Smith (1995). The diagonal elements of the covariance matrix of θ's are the variances. Obviously, in the case of a single θ, the covariance matrix is a scalar. The standard errors of the means of θ's are, simply, the square roots of variances divided by the square root of the number of observations.

Parameter estimates and standard errors are reported in the tables of the results in section 5 and used in t-tests to test the hypothesis that the models for the home-to-work and the reverse trip chains are different. An additional hypothesis would be to test whether we need a different model in each stage of the trip chain. If the values of θ's are substantially different and the standard errors relatively small, we may be able to distinguish between the two models under consideration.

5.3 Goodness of Fit

Assuming that observations N_{wk} are independently Poisson distributed, the (Pearson) χ^2 statistic,

$$\chi^2 = \sum_{wk} \frac{(N_{wk} - \hat{T}_{wk})^2}{\hat{T}_{wk}} \tag{24}$$

where $\hat{T}_{wk}$ is an estimate of T_{wk}, is an appropriate measure of the overall fit of a model. For large T_{wk}'s the χ^2 statistic has the χ^2 distribution. This will not be true in our case where T_{wk}'s are very small. We choose, however, to adopt the particular notation for the χ^2 statistic for its elegance.

In the case of Poisson gravity models, $\hat{T}_{wk} \approx T_{wk}$. This is because of the existence of large enough (even though individual N_{wk}'s may be small, or even zero) sufficient statistics $\sum_k N_{wk}$, $\sum_w N_{wk}$, and $\sum_{wk} N_{wk}c_{wk}$ which enter into the computation of the normal equations. If $\hat{T}_{wk} \approx T_{wk}$, then $\chi^2 = Z^2$, where

$$Z^2 = \sum_{wk} \frac{(N_{wk} - T_{wk})^2}{T_{wk}} \tag{25}$$

Since $E(N_{wk}) = T_{wk}$ and because N_{wk} have the Poisson distribution,

$$\text{var}(N_{wk}) = E(N_{wk} - T_{wk})^2 = T_{wk} \tag{26}$$

Therefore, $E(Z^2) = WK$, where W the number of origin-final destination zonal pairs w and K the number of intermediate destinations k. Equivalently, $E(Z^2/WK) = 1$. Thus the so-called "χ^2-ratio" (appearing also in the tables of section 5) has an expectation which is asymptotically 1. It can be shown (Kim, 1993; Sen and Smith, 1995, Chapter 5) that the variance of the χ^2-ratio is,

$$\text{var}[Z^2/(WK)] = \sum_{wk}[(T_{wk}W^2K^2)^{-1} + 2(WK)^{-2}] \tag{27}$$

If T_{wk}'s are bounded away from zero (which is the case in exponential gravity models with finite parameters θ), the variance of $Z^2/WK \to 0$, as $WK \to \infty$. This result can be seen in the tables of section 5, although neither W nor K are very large.

5.4 Empirical Results

A variety of functional specifications for separation profiles were tested, based on airline distance or travel time as separation measures, and a number of different gravity models were estimated. The results are shown in Tables I-X for two gravity models at a time. Tables I to V are pertinent to the home-to-work single-stop work trip chains, while Tables VI to X to the work to home single-stop work trip chains.

In particular, Table I (and VI) presents two specifications with one separation measure (distance or travel time). The first model considers the sum of separation measures at each stage of the trip chain. In the second model the direct home to work distance (travel time) is subtracted from the previous sum. In Table II (and VII) we continue to consider only one separation measure. This time, however, the square roots of the respective separation measures in Table I are considered.

In Table III (and VIII) two models are specified using two separation measures. The first model considers distances (or travel time) of each leg of the trip. In the second model the square roots of the separation measures of each leg are used. Table IV (and IX) the two models consider combinations of linear and the square-root transformation of distances and travel times at each stage of the trip. Finally, in Table V (and X) the two gravity models consider combinations of linear and the logarithmic transformation of distances and travel times at each stage of the trip.

The overall fit of each of the twenty models considered in Tables I to X is very good. Typical examples of estimated frequencies are given in Table XII (for the home-to-work trip chain) and Table XIV (for the reverse trip chain). Comparisons between the cell-to-cell frequencies of the corresponding estimated and observed trip tables leave little doubt that the models presented in Tables I-X are good starting points for modeling single-stop work trip chains.

In most of the cases the χ^2-ratio remains lower than 1 with variance equal to zero. This may be due to the fact that the small number of w's and k's does not compensate for the large number of zero-valued N_{wk}'s. This phenomenon for N_{wk} matrices containing mainly zeros has also been noticed by Boyle and Flowerdew (1993), who suggest to not spare the effort of searching for other relevant variables to make the model fit better, despite an early success of a very good fit.

Under the Poisson assumption and with N_{wk}'s sufficiently large, the χ^2-ratio would be approximately 1. Thus, if the model in (17) fits perfectly and each of the separation measures, be it distance or travel time, was chosen perfectly, then the χ^2-ratio would be close to 1. However, since the data are actual observations we had expected that the χ^2-ratio would be larger than 1. This is because not only the sequencing of activities in a trip chain is interdependent, but also trips at each stage of a trip chain are interdependent. For example, at the time the 1990 CATS Household Travel Survey was conducted, a nationwide average of 1.51 person-trips for each privately owned vehicle was observed (Vincent *et al*, 1994). This alone would raise the expectation of the χ^2-ratio to 1.51. Therefore, even those few models with a χ^2-ratio overall measure of fit larger than 1 are not too bad. Note also, that travel time alone gives almost uniformly higher χ^2-ratios than distances used in the same functional specification.

Another issue which needs further investigation is whether commuters perceive separation measures such as travel time linearly or non-linearly. It is not conclusive from this study whether square roots of travel time lead to better model fits than travel time alone. In addition, with such large standard errors for parameters estimates (larger in the case of home-to-work trip chains due to an even smaller sample size) the usual t-test cannot distinguish at any reasonable significance level between models using distance or travel time only. This apparent weakness can be turned into an advantage if we think of the models developed in this study as stand-alone models which can be applied successfully in situations where more detailed data is difficult to obtain.

More importantly, given that travelers trade off different separation measures such as travel time and travel cost, it would be interesting to test whether these trade-offs are perceived in a linear or nonlinear fashion. We think, however, that these issues can be better addressed in a more general framework where the trip-chain models developed in this study are an integral component.

An interesting point related also to the large standard errors is that it may not be necessary to estimate different parameters at each stage of the trip chain given the same separation functional specification at each stage. This result can be seen in Tables III and VIII where travel costs are assumed to be perceived either linearly in both stages or non-linearly in both stages. This is a pleasant result from a modeling perspective because models with fewer parameters are easier to built, maintain and transfer. We will stop short, however, from making any strong recommendation.

Another issue may be seen to arise from Tables IV, V, IX and X. The models reported in those tables have a different functional specification for travel costs at each stage of the trip chain. Sen and Smith (1995, Chapter 5) report that the *power-exponential* specifications in Tables V and X in particular, are advantageous from a statistical viewpoint when comparing the relative benefits of power specifications and exponential specifications (in our case the travel costs are stage-specific). In addition, the power-exponential specification may have behavioral meaning in certain cases (e.g. "marginal perceived cost" as in Zaryouni and Liebman, 1976).

A note on the sensitivity of the parameters at each stage of the trip chain will now be made. In the models built with the same functional specification for travel costs, the parameter estimates for the first stage of the trip, seem to have a higher (absolute) value. This is more obvious for the home-to-work than the reverse trip chain and means that the home location "pulls" stronger than the work location when deciding to make a stop. This is in agreement with the findings in Kim (1993) for shopping trip chains.

Finally and most importantly, in comparing the same models for the home-to-work and the reverse trip chains a critical result appears to emerge. Parameter estimates for the same stage of the trip for those two types of trip chains do not seem to be distinguishable. At this point we prefer to remain cautious not to generalize, not only because of sample sizes, but also because further investigation is needed for trip chains with more than two stages. However, we feel that if this result holds, it can be a very fortunate coincidence for the transportation modeler.

TABLE I

HOME TO WORK
ONE LINEAR SEPARATION MEASURE

Parameters	$c_{ik} + c_{kj}$		$c_{ik} + c_{kj} - c_{ij}$	
	Distance	Travel Time	Distance	Travel Time
θ	-0.1625	-0.0920	-0.1615	-0.0918
s.e.(θ)	0.0335	0.0285	0.0332	0.0284
χ^2-ratio	0.42	0.57	0.41	0.57
var(χ^2)	0.00	0.00	0.00	0.00
$T_{wk} =$	$A_w B_k \exp[\theta(c_{ik} + c_{kj})]$		$A_w \exp(-\theta c_{ij}) B_k \exp[\theta(c_{ik} + c_{kj})]$	

TABLE II

HOME TO WORK
SQUARE ROOT OF ONE SEPARATION MEASURE

Parameters	$\sqrt{c_{ik} + c_{kj}}$		$\sqrt{c_{ik} + c_{kj} - c_{ij}}$	
	Distance	Travel Time	Distance	Travel Time
θ	-1.5502	-1.1807	-1.0995	-0.8535
s.e.(θ)	0.3297	0.2328	0.2266	0.1582
χ^2-ratio	1.19	1.26	1.16	1.26
var(χ^2)	0.00	0.00	0.00	0.00
$T_{wk} =$	$A_w B_k \exp(\theta\sqrt{c_{ik} + c_{kj}})$		$A_w B_k \exp(\theta\sqrt{c_{ik} + c_{kj} - c_{ij}})$	

TABLE III

HOME TO WORK: TWO LINEAR OR
SQUARE ROOTS OF TWO SEPARATION MEASURES

Parameters	$(c_{ik}, c_{kj})'$ Distance	$(c_{ik}, c_{kj})'$ Travel Time	$(\sqrt{c_{ik}}, \sqrt{c_{kj}})'$ Distance	$(\sqrt{c_{ik}}, \sqrt{c_{kj}})'$ Travel Time
θ_1	-0.2053	-0.1186	-1.1648	-0.9337
s.e.(θ_1)	0.0421	0.0383	0.2032	0.1735
θ_2	-0.1036	-0.0615	-0.7726	-0.5884
s.e.(θ_2)	0.0215	0.0178	0.1276	0.1017
χ^2-ratio	0.38	0.59	0.42	1.31
var(χ^2)	0.00	0.00	0.00	0.00
$T_{wk} =$	$A_w B_k \exp(\theta_1 c_{ik} + \theta_2 c_{kj})$		$A_w B_k \exp(\theta_1 \sqrt{c_{ik}} + \theta_2 \sqrt{c_{kj}})$	

TABLE IV

HOME TO WORK: LINEAR-SQUARE ROOT
COMBINATION OF TWO SEPARATION MEASURES

Parameters	$(c_{ik}, \sqrt{c_{kj}})'$ Distance	$(c_{ik}, \sqrt{c_{kj}})'$ Travel Time	$(\sqrt{c_{ik}}, c_{kj})'$ Distance	$(\sqrt{c_{ik}}, c_{kj})'$ Travel Time
θ_1	-0.1973	-0.1144	-1.2085	-0.9625
s.e.(θ_1)	0.0411	0.0278	0.2158	0.1823
θ_2	-0.7738	-0.6157	-0.1002	-0.0576
s.e.(θ_2)	0.1281	0.1152	0.0208	0.0117
χ^2-ratio	0.36	0.85	0.38	0.83
var(χ^2)	0.00	0.00	0.00	0.00
$T_{wk} =$	$A_w B_k \exp(\theta_1 c_{ik} + \theta_2 \sqrt{c_{kj}})$		$A_w B_k \exp(\theta_1 \sqrt{c_{ik}} + \theta_2 c_{kj})$	

TABLE V

HOME TO WORK: LINEAR-LOGARITHMIC
COMBINATION OF TWO SEPARATION MEASURES

Parameters	$(c_{ik}, \log c_{kj})'$		$(\log c_{ik}, c_{kj})'$	
	Distance	Travel Time	Distance	Travel Time
θ_1	-0.1951	-0.1146	-1.5396	-1.6912
s.e.(θ_1)	0.0397	0.0325	0.2528	0.2727
θ_2	-1.0999	-1.1486	-0.1110	-0.0597
s.e.(θ_2)	0.1425	0.1518	0.0308	0.0153
χ^2-ratio	0.42	1.51	0.45	1.62
var(χ^2)	0.00	0.00	0.00	0.00
$T_{wk} =$	$A_w B_k (c_{kj})^{\theta_2} \exp(\theta_1 c_{ik})$		$A_w B_k (c_{ik})^{\theta_1} \exp(\theta_2 c_{kj})$	

TABLE VI

WORK TO HOME
ONE LINEAR SEPARATION MEASURE

Parameters	$c_{ik} + c_{kj}$		$c_{ik} + c_{kj} - c_{ij}$	
	Distance	Travel Time	Distance	Travel Time
θ	-0.2247	-0.1158	-0.2257	-0.1167
s.e.(θ)	0.0312	0.221	0.0314	0.225
χ^2-ratio	0.46	0.77	0.47	0.78
var(χ^2)	0.00	0.00	0.00	0.00
$T_{wk} =$	$A_w B_k \exp[\theta(c_{ik} + c_{kj})]$		$A_w \exp(-\theta c_{ij}) B_k \exp[\theta(c_{ik} + c_{kj})]$	

TABLE VII

WORK TO HOME
SQUARE ROOT OF ONE SEPARATION MEASURE

Parameters	$\sqrt{c_{ik}+c_{kj}}$ Distance	$\sqrt{c_{ik}+c_{kj}}$ Travel Time	$\sqrt{c_{ik}+c_{kj}-c_{ij}}$ Distance	$\sqrt{c_{ik}+c_{kj}-c_{ij}}$ Travel Time
θ	-1.5917	-1.1827	-1.1152	-0.8825
s.e.(θ)	0.2989	0.2485	0.2323	0.1132
χ^2-ratio	1.10	1.10	1.10	1.10
var(χ^2)	0.00	0.00	0.00	0.00
$T_{wk} =$	$A_w B_k \exp(\theta\sqrt{c_{ik}+c_{kj}})$		$A_w B_k \exp(\theta\sqrt{c_{ik}+c_{kj}-c_{ij}})$	

TABLE VIII

WORK TO HOME: TWO LINEAR OR
SQUARE ROOTS OF TWO SEPARATION MEASURES

Parameters	$(c_{ik}, c_{kj})'$ Distance	$(c_{ik}, c_{kj})'$ Travel Time	$(\sqrt{c_{ik}}, \sqrt{c_{kj}})'$ Distance	$(\sqrt{c_{ik}}, \sqrt{c_{kj}})'$ Travel Time
θ_1	-0.2275	-0.1130	-1.2664	-0.8607
s.e.(θ_1)	0.0412	0.0228	0.2335	0.1107
θ_2	-0.2224	-0.1184	-1.3445	-0.9639
s.e.(θ_2)	0.0398	0.0230	0.2447	0.1225
χ^2-ratio	0.46	0.79	0.29	0.44
var(χ^2)	0.00	0.00	0.00	0.00
$T_{wk} =$	$A_w B_k \exp(\theta_1 c_{ik} + \theta_2 c_{kj})$		$A_w B_k \exp(\theta_1\sqrt{c_{ik}} + \theta_2\sqrt{c_{kj}})$	

TABLE IX

WORK TO HOME: LINEAR-SQUARE ROOT COMBINATION OF TWO SEPARATION MEASURES

Parameters	$(c_{ik}, \sqrt{c_{kj}})'$		$(\sqrt{c_{ik}}, c_{kj})'$	
	Distance	Travel Time	Distance	Travel Time
θ_1	-0.2035	-0.1011	-1.3265	-0.9069
s.e.(θ_1)	0.0393	0.0215	0.2518	0.1232
θ_2	-1.4376	-0.9967	-0.1934	-0.1087
s.e.(θ_2)	0.2858	0.1328	0.0335	0.0229
χ^2-ratio	0.30	0.48	0.43	0.61
var(χ^2)	0.00	0.00	0.00	0.00
$T_{wk} =$	$A_w B_k \exp(\theta_1 c_{ik} + \theta_2 \sqrt{c_{kj}})$		$A_w B_k \exp(\theta_1 \sqrt{c_{ik}} + \theta_2 c_{kj})$	

TABLE X

WORK TO HOME: LINEAR-LOGARITHMIC COMBINATION OF TWO SEPARATION MEASURES

Parameters	$(c_{ik}, \log c_{kj})'$		$(\log c_{ik}, c_{kj})'$	
	Distance	Travel Time	Distance	Travel Time
θ_1	-0.1880	-0.0934	-1.4417	-1.4043
s.e.(θ_1)	0.0383	0.0195	0.2767	0.2629
θ_2	-1.7038	-1.6009	-0.1760	-0.1024
s.e.(θ_2)	0.3207	0.3017	0.0364	0.0205
χ^2-ratio	0.28	0.39	0.62	0.63
var(χ^2)	0.00	0.00	0.00	0.00
$T_{wk} =$	$A_w B_k (c_{kj})^{\theta_2} \exp(\theta_1 c_{ik})$		$A_w B_k (c_{ik})^{\theta_1} \exp(\theta_2 c_{kj})$	

5.5 Short-Term Forecasting

Let the random variable $N_{wk}^{(f)}$ be a future observation of the flow from w to k. In the short run, $N_{wk}^{(f)}$ and N_{wk} will be highly (serially) correlated. In the case of commuter trips, for example, both $N_{wk}^{(f)}$ and N_{wk} will be counts of mostly the same people stopping at the same intermediate destinations. This assumption appears to be very strong from a first point of view. We will argue immediately, however, that this is not the case assuming implicitly that the separation configuration will not change in a manner that will have a drastic impact on activity patterns during the forecast period.

Home and work locations do not change often and may be assumed to remain the same in the short run (e.g. a five-year period). We will make now an assumption which is consistent with those made in the presentation of the threshold theory. In particular, we will assume that individual threshold considerations remain relatively stable over that short period of time, so that the sizes of the populations of commuters and opportunities in intermediate destinations are not influenced by factors which are not consistent with Poisson randomness. Then for activities in intermediate destinations three cases may be considered.

1. The first case can be made for discretionary activities for services which, by nature, stay at the same location for longer periods of time, such as shopping malls, banks, theaters etc. In those cases there is no reason to believe (at least not without additional information) that commuters will change attitudes toward patronizing the same facilities in the near future. Hence commuting patterns will remain the same.

2. The second case relates to discretionary activities for services which are more "foot-loose", such as visits to dry-cleaner's, fast-food restaurants, super markets, etc. If these services move to other locations during the forecast period, then we will assume that individual threshold considerations toward separation and accessibility (see Metaxatos, 1995 for precise definitions) may prevent the realization of a radically changed behavior. Hence we assume that individuals will choose to pursue the same activities in nearby locations. Therefore, if the spatial configurations of activities in intermediate destinations have been designed to be sufficiently homogeneous, commuting patterns to those destinations will remain relatively the same.

3. Finally, the third case relates to mandatory (in the relative sense adopted in previous chapters) intermediate activities, such as visits to a doctor, a friend, etc. Of course, one may argue that trips for this type of activities are more likely to be made directly to the relevant destinations and not as part of trip

chains. If there is enough evidence to support this claim, then there would be no difficulty in forecasting those trips using more traditional methods. If, however, we think of these activities as part of future trip chains, then we argue that the more behavioral threshold theory framework may accommodate even this type of behavior.

A future move of the family doctor, or a personal friend will almost certainly imply a change in individual trip-chaining patterns, because individual attitudes toward emissivity threshold considerations (see Metaxatos, 1995) will come into the picture in a more dramatic fashion than separation or accessibility threshold considerations. We choose to believe, however, that those cases can be treated as individual uncommon cases which will not affect overall patterns of intermediate stops.

Given the discussion above we may then argue that the variance of the difference of future and present observations will be smaller than the variance of future observations alone. Hence,

$$\mathrm{var}(N_{wk}^{(f)} - N_{wk}) = \mathrm{var}(N_{wk}^{(f)}) + \mathrm{var}(N_{wk}) - 2\mathrm{Cov}(N_{wk}^{(f)} N_{wk}) \quad (28)$$

where, var(.) stands for the "variance of" and Cov(.) for the "covariance of", will be small. In such cases, especially if N_{wk}'s are known, Sen and Smith (1995, Chapter 5) suggest that it would be preferable to predict $(N_{wk}^{(f)} - N_{wk})$'s and add the predictions to the N_{wk}'s. A possible set of predictions for $(N_{wk}^{(f)} - N_{wk})$'s are $(\hat{T}_{wk}^{(f)} - \hat{T}_{wk})$'s. Thus, if the future is not too far off,

$$N_{wk} + (\hat{T}_{wk}^{(f)} - \hat{T}_{wk}) \quad (29)$$

may yield a better prediction than $\hat{T}_{wk}$. The computation of $\Delta\hat{T}_{wk} = \hat{T}_{wk}^{(f)} - \hat{T}_{wk}$ can be made easily using the LDSF procedure (see Sen and Smith, 1995, Chapter 5).

6 Conclusion

We have argued that the assumption on the decision process underlying trip-chaining behavior is instrumental in the development of trip-chain models. In particular, the assumption of an *r-sequential* decision making process legitimizes the study of single-stop trip chains. The threshold theory for single-stop trip chains operationalizes an alternative framework for *r-sequential trip-chaining processes* focusing on the individual traveler in an aggregate way. As such, the theory may be seen to operate in the range between aggregate and disaggregate models of travel behavior. The basic implication of the theory is the Poisson nature of the resulting trip-chain frequencies. Trip chains and, in particular work trip chains, may be seen as commuting trip patterns disrupted for single discretionary activities. If individual commuters choose their intermediate activities so that mean frequencies are comparable to their variance, then the threshold theory provides an appealing alternative to modeling *r-sequential trip-chaining behavior.*

The empirical analysis has corroborated the belief that the theoretical framework proposed for modeling trip chains has initiated a legitimate and promising research direction. However, the empirical work done in this thesis is by no means exhaustive. Hence, additional separation measures and combinations of separation measures need to be tested. Moreover, in cases where emissivity and accessibility threshold considerations are too obvious to ignore, then we need to explore additional functional specifications for models proposed in Metaxatos (1995). We anticipate that on-going research will focus on the following issues: 1) What additional specifications for threshold variables is needed for a better model fit and lower standard errors? 2) How do commuters perceive these threshold variables? 3) How can the theoretical framework be extended to consider multi-stop and mode chaining and time? 4) What interaction effects between trip-chaining behavior and land use patterns are implied? 5) In which cases contagion effects or band-wagon effects which lead individual commuters to identical interaction choices, require the consideration of additional trip-chaining models of restricted or extra variation? 6) Do individual threshold considerations remain relatively stable over short periods of time? 7) How can congestion effects on trip-chaining patterns (and the other way round) be studied in the light of these new developments? We would like, therefore, to invite those who have become interested from the initial success of the new development to offer additional insights.

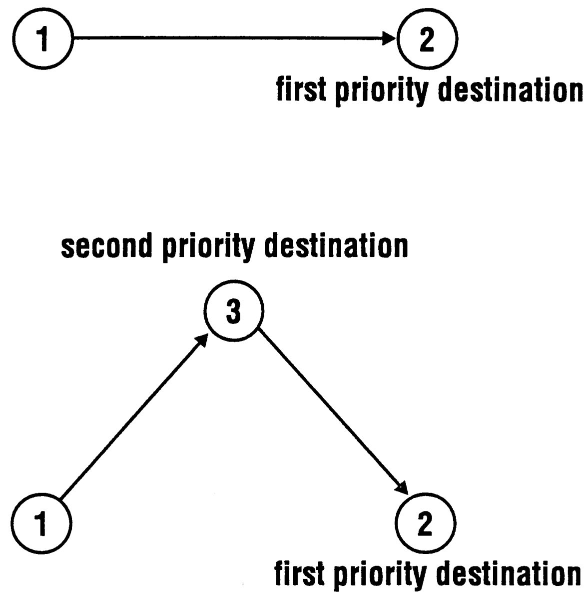
1
2
first priority destination
second priority destination
3
1
2
first priority destination

Fig 4

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TABLE XI

HOME-TO-WORK
OBSERVED TRIPS

ij-pairs	k-destinations 1	2	3	4	5
1	1	0	0	0	0
2	1	0	0	0	0
3	1	0	0	0	0
4	1	0	0	0	0
5	2	0	0	0	0
6	1	0	0	0	0
7	1	0	0	0	0
8	1	0	0	0	0
9	0	1	0	0	0
10	1	0	0	0	0
11	1	0	0	0	0
12	1	0	0	0	0
13	0	0	1	0	0
14	0	0	1	0	0
15	0	1	0	0	0
16	0	1	0	0	0
17	0	1	0	0	0
18	0	1	0	0	0
19	0	2	0	0	0
20	0	1	0	0	0
21	0	1	0	0	0
22	0	1	0	0	0
23	0	0	1	0	0
24	0	0	1	0	0
25	0	0	2	0	0
26	0	0	0	0	1
27	0	0	1	0	0
28	0	0	1	0	0
29	0	0	1	0	0
30	0	0	0	0	1
31	0	0	0	1	0
32	0	0	0	6	0
33	0	0	0	1	0
34	0	0	0	1	0
35	0	0	0	2	0

TABLE XI

HOME-TO-WORK
OBSERVED TRIPS
(CONTINUED)

ij-pairs	k-destinations				
	1	2	3	4	5
36	0	0	0	0	1
37	0	0	0	0	1
38	0	0	0	0	1
39	0	0	0	0	1
40	0	0	0	0	1
41	0	0	0	0	2
42	0	0	0	0	1
43	0	0	0	0	1
44	0	0	0	0	2
45	0	0	0	0	2
46	0	0	0	0	1
47	0	0	0	1	0
48	0	0	0	0	1
49	0	0	1	0	0
50	0	0	1	0	0
51	1	0	0	0	0

TABLE XII

HOME-TO-WORK ESTIMATED TRIPS

ij-pairs	k-destinations				
	1	2	3	4	5
1	1.0056573	0.0000619	0.0000128	0.0000001	0.0000005
2	1.0048568	0.0004972	0.0003209	0.0000023	0.0000155
3	1.0043260	0.0007564	0.0005607	0.0000033	0.0000214
4	1.0057078	0.0000227	0.0000050	0.0000000	0.0000002
5	2.0109157	0.0003638	0.0001572	0.0000010	0.0000061
6	1.0040187	0.0010976	0.0005042	0.0000032	0.0000182
7	0.9999905	0.0037785	0.0015226	0.0000170	0.0001049
8	1.0003323	0.0045063	0.0005167	0.0000034	0.0000149
9	0.7522021	0.0513065	0.1818613	0.0018202	0.0112587
10	0.3109930	0.6383741	0.0058771	0.0000517	0.0001200
11	0.8801282	0.1121150	0.0044331	0.0000403	0.0001199
12	0.8669945	0.1217534	0.0070103	0.0000716	0.0002009
13	0.0464601	0.4575182	(.4493346	0.0023233	0.0055271
14	0.0534868	0.2886405	0.6054151	0.0049199	0.0193068
15	0.0018905	0.9303545	0.0002980	0.0000054	0.0000075
16	0.0151135	0.9154366	0.0030174	0.0000420	0.0000688
17	0.0057256	0.9237019	0.0034654	0.0000500	0.0000770
18	0.0012558	0.9306926	0.0005509	0.0000108	0.0000146
19	0.0010902	1.8631817	0.0006172	0.0000118	0.0000159
20	0.0182573	0.8909463	0.0251896	0.0002645	0.0005296
21	0.0021468	0.9296291	0.0007957	0.0000127	0.0000194
22	0.0011846	0.9305332	0.0007823	0.0000131	0.0000195
23	0.0000521	0.0001655	0.8075664	0.0300821	0.1562838
24	0.0000506	0.0001158	0.8809947	0.0146210	0.0952300
25	0.0000487	0.0000910	1.9574423	0.0026137	0.0134576
26	0.0000214	0.0000505	0.9611635	0.0045106	0.0218325
27	0.0008732	0.0033072	0.9478232	0.0086061	0.0271808
28	0.0000645	0.0003483	0.9681649	0.0051044	0.0135620
29	0.0000283	0.0002941	0.9539542	0.0146521	0.0189208
30	0.0000014	0.0000014	0.0094480	0.0304903	0.9886245
31	0.0000025	0.0000135	0.0915191	0.7358050	0.1966988
32	0.0000003	0.0000024	0.0041955	6.0358870	0.1253714
33	0.0000002	0.0000014	0.0030592	0.8795831	0.1450075
34	0.0000002	0.0000011	0.0017269	0.9588229	0.0670468
35	0.0000005	0.0000028	0.0053105	1.7396576	0.3103936

TABLE XII

HOME-TO-WORK ESTIMATED TRIPS (CONTINUED)

	k-destinations				
ij-pairs	1	2	3	4	5
36	0.0000003	0.0000009	0.0035894	0.0845477	0.9406042
37	0.0000007	0.0000022	0.0067464	0.4018611	0.6195513
38	0.0000014	0.0000027	0.0229983	0.0467052	0.9582512
39	0.0000009	0.0000015	0.0110777	0.0316801	0.9857335
40	0.0000016	0.0000024	0.0190133	0.0323118	0.9768215
41	0.0000010	0.0000020	0.0099648	0.1566692	1.8907451
42	0.0000002	0.0000003	0.0015227	0.0107863	1.0166255
43	0.0000055	0.0000050	0.0217190	0.0350940	0.9712062
44	0.0000071	0.0000091	0.0639721	0.0719008	1.9192832
45	0.0000026	0.0000038	0.0261304	0.0567087	1.9739798
46	0.0000010	0.0000015	0.0100226	0.0261719	0.9923500
47	0.0000003	0.0000010	0.0038832	0.3939483	0.6304635
48	0.0000011	0.0000022	0.0114150	0.1742442	0.8426169
49	0.0003306	0.0002258	0.9359758	0.0050408	0.0470649
50	0.0000577	0.0000555	0.9673480	0.0022266	0.0176258
51	1.0057097	0.0000212	0.0000048	0.0000000	0.0000002

TABLE XIII

OBSERVED WORK-TO-HOME TRIPS

ij-pairs	k-destinations						
	1	2	3	4	5	6	7
1	1	0	0	0	0	0	0
2	0	1	0	0	0	0	0
3	1	0	0	0	0	0	0
4	1	0	0	0	0	0	0
5	1	0	0	0	0	0	0
6	1	0	0	0	0	0	0
7	1	0	0	0	0	0	0
8	1	0	0	0	0	0	0
9	0	1	0	0	0	0	0
10	1	0	0	0	0	0	0
11	1	0	0	0	0	0	0
12	0	1	0	0	0	0	0
13	0	1	0	0	0	0	0
14	0	1	0	0	0	0	0
15	0	1	0	0	0	0	0
16	0	0	1	0	0	0	0
17	0	0	1	0	0	0	0
18	0	0	2	0	0	0	0
19	0	0	1	0	0	0	0
20	0	0	1	0	0	0	0
21	0	0	0	1	0	0	0
22	0	0	1	0	0	0	0
23	0	0	1	0	0	0	0
24	0	0	0	0	1	0	0
25	0	0	0	0	0	0	1
26	0	0	0	1	0	0	0
27	0	0	0	1	0	0	0
28	0	0	0	1	0	0	0
29	0	0	0	1	0	0	0
30	0	0	0	1	0	0	0
31	0	0	0	1	0	0	0
32	0	0	0	1	0	0	0
33	0	0	1	0	0	0	0
34	0	0	0	1	0	0	0
35	0	0	0	0	0	1	0

TABLE XIII

OBSERVED WORK-TO-HOME TRIPS (CONTINUED)

	k-destinations						
ij-pairs	1	2	3	4	5	6	7
36	0	0	0	0	0	0	1
37	0	0	0	0	0	1	0
38	0	0	0	0	0	1	0
39	0	0	0	0	1	0	0
40	0	0	0	1	0	0	0
41	0	0	0	0	0	3	0
42	0	0	0	0	1	0	0
43	0	0	0	0	2	0	0
44	0	0	1	0	0	0	0
45	0	0	0	0	0	0	0
46	0	0	0	0	0	1	0
47	0	0	0	0	3	0	0
48	0	0	0	0	0	1	0
49	0	0	0	0	1	0	0
50	0	1	0	0	0	0	0
51	0	0	0	1	0	0	0
52	0	0	0	0	0	1	0
53	0	0	0	0	0	3	0
54	0	0	0	0	0	1	0
55	0	0	0	0	1	0	0
56	0	1	0	0	0	0	0
57	0	0	0	0	0	1	0
58	0	0	0	0	0	1	0
59	1	0	0	0	0	0	0
60	0	1	0	0	0	0	0
61	0	0	0	0	0	0	1
62	0	0	1	0	0	0	0
63	1	0	0	0	0	0	0
64	0	0	0	0	0	0	1
65	0	1	0	0	0	0	0
66	0	0	1	0	0	0	0
67	0	0	0	0	0	0	1
68	0	0	0	0	0	0	1
69	0	0	0	0	0	0	1
70	0	0	1	0	0	0	0

TABLE XIII

OBSERVED WORK-TO-HOME
TRIPS (CONTINUED)

	k-destinations						
ij-pairs	1	2	3	4	5	6	7
71	0	0	0	0	0	1	0
72	0	0	0	0	0	1	0
73	0	0	0	0	0	0	1
74	0	0	0	0	0	0	1
75	0	0	0	0	0	1	0
76	0	0	0	0	0	0	1
77	0	0	0	0	0	0	1
78	0	0	0	0	0	0	1
79	0	0	0	0	0	1	0
80	0	0	0	0	0	1	0

TABLE XIV

ESTIMATED WORK-TO-HOME TRIPS

ij-pairs	k-destinations						
	1	2	3	4	5	6	7
1	0.367955	0.609277	0.003409	0.000143	0.000076	0.000102	0.000806
2	0.815217	0.160213	0.003072	0.000224	0.000198	0.000177	0.000709
3	0.913542	0.064628	0.000867	0.000055	0.000047	0.000040	0.000169
4	0.954564	0.024334	0.000203	0.000010	0.000008	0.000007	0.000035
5	0.279236	0.165643	0.060234	0.058712	0.222670	0.190036	0.024317
6	0.905074	0.073661	0.000494	0.000023	0.000015	0.000016	0.000098
7	0.943488	0.035266	0.000345	0.000019	0.000015	0.000013	0.000065
8	0.923116	0.055234	0.000720	0.000044	0.000040	0.000031	0.000119
9	0.817818	0.157087	0.003622	0.000268	0.000303	0.000181	0.000527
10	0.934318	0.044425	0.000390	0.000020	0.000016	0.000014	0.000068
11	0.218725	0.184851	0.115686	0.202259	0.158348	0.091947	0.026616
12	0.082307	0.898692	0.001614	0.000054	0.000028	0.000034	0.000275
13	0.012783	0.958936	0.009853	0.000411	0.000291	0.000213	0.000894
14	0.009168	0.964219	0.007028	0.000261	0.000100	0.000193	0.002405
15	0.009178	0.914931	0.053381	0.000892	0.000351	0.000449	0.004469
16	0.001011	0.035827	0.822063	0.007222	0.001396	0.003739	0.117023
17	0.000225	0.007836	0.967574	0.002475	0.000548	0.000873	0.008845
18	0.000132	0.004590	1.955012	0.002675	0.000483	0.000936	0.012903
19	0.000234	0.007324	0.865457	0.005278	0.000827	0.002640	0.106596
20	0.001055	0.026930	0.902344	0.012264	0.003656	0.004571	0.037827
21	0.000214	0.005580	0.583611	0.100956	0.008132	0.059390	0.233922
22	0.000196	0.006202	0.553214	0.007893	0.001035	0.003961	0.415529
23	0.000035	0.001065	0.973265	0.002698	0.000362	0.000834	0.010147
24	0.000061	0.001117	0.066451	0.037778	0.869373	0.035480	0.017833
25	0.000151	0.003696	0.529845	0.135795	0.008753	0.039379	0.274255
26	0.000036	0.000677	0.042358	0.856061	0.028613	0.066253	0.012971
27	0.000030	0.000581	0.035550	0.851838	0.022859	0.080251	0.015907
28	0.000015	0.000285	0.015987	0.910598	0.016627	0.056968	0.006711
29	0.000030	0.000566	0.032035	0.824502	0.023700	0.108532	0.017963
30	0.000027	0.000526	0.030389	0.816750	0.017072	0.118056	0.024329
31	0.000008	0.000135	0.005062	0.875394	0.050094	0.075658	0.002186
32	0.000003	0.000061	0.003018	0.947441	0.005511	0.048949	0.002180
33	0.000201	0.004734	0.586434	0.249604	0.016213	0.078483	0.059961
34	0.000008	0.000148	0.007320	0.871115	0.009402	0.112844	0.006848
35	0.000003	0.000052	0.001893	0.085641	0.008096	0.917363	0.002294

TABLE XIV

ESTIMATED WORK-TO-HOME TRIPS (CONTINUED)

ij-pairs	*k*-destinations						
	1	2	3	4	5	6	7
36	0.000153	0.003649	0.150227	0.198547	0.010218	0.092939	0.538448
37	0.000002	0.000036	0.001237	0.064026	0.006843	0.941885	0.001537
38	0.000007	0.000129	0.005693	0.755290	0.014333	0.228350	0.005153
39	0.000003	0.000054	0.001302	0.071050	0.873249	0.083829	0.000733
40	0.000007	0.000120	0.003610	0.343209	0.147625	0.518154	0.002504
41	0.000004	0.000069	0.002207	0.201726	0.022583	2.817906	0.002265
42	0.000040	0.000462	0.004237	0.010860	1.006133	0.010513	0.000699
43	0.018754	0.305801	1.221458	0.169976	0.141974	0.059332	0.068505
44	0.002637	0.052438	0.148855	0.021079	0.003399	0.018086	0.741560
45	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
46	0.000004	0.000074	0.002377	0.043089	0.006982	0.958290	0.004818
47	0.000000	0.000003	0.000048	0.000858	3.098101	0.001932	0.000018
48	0.000002	0.000028	0.000573	0.016926	0.556362	0.451106	0.000388
49	0.000001	0.000012	0.000206	0.004332	1.002201	0.026223	0.000112
50	0.008174	0.356213	0.127422	0.123623	0.191650	0.166161	0.028373
51	0.000007	0.000104	0.002521	0.071780	0.133317	0.808500	0.001375
52	0.000001	0.000025	0.000635	0.031680	0.047884	0.935877	0.000512
53	0.000000	0.000008	0.000231	0.011094	0.004364	3.032387	0.000276
54	0.000005	0.000093	0.002570	0.036884	0.007212	0.962630	0.006255
55	0.000000	0.000005	0.000076	0.001308	1.027455	0.004671	0.000033
56	0.538668	0.401582	0.024297	0.002105	0.001172	0.002074	0.011425
57	0.000256	0.003109	0.013054	0.040239	0.015135	0.923600	0.019561
58	0.000132	0.001595	0.008708	0.035354	0.014185	0.940925	0.014413
59	0.432540	0.445396	0.050267	0.006980	0.004237	0.009113	0.033960
60	0.709933	0.256053	0.009554	0.000486	0.000227	0.000383	0.003694
61	0.001969	0.036052	0.131487	0.021565	0.003453	0.019793	0.773837
62	0.239737	0.562376	0.100619	0.011167	0.008424	0.010783	0.050904
63	0.774128	0.202261	0.002625	0.000121	0.000065	0.000089	0.000687
64	0.064092	0.745972	0.090052	0.005293	0.001814	0.005179	0.071765
65	0.009216	0.939889	0.029630	0.000569	0.000217	0.000311	0.003678
66	0.007366	0.220121	0.525294	0.017926	0.005142	0.010568	0.201298
67	0.000279	0.005927	0.078374	0.036801	0.005380	0.061394	0.801633
68	0.000171	0.004493	0.083776	0.007761	0.001050	0.006620	0.883595
69	0.000169	0.004232	0.082983	0.010566	0.001443	0.010249	0.878000
70	0.000313	0.005624	0.072078	0.089962	0.018003	0.286228	0.525631

TABLE XIV

ESTIMATED WORK-TO-HOME TRIPS (CONTINUED)

ij-pairs	k-destinations						
	1	2	3	4	5	6	7
71	0.000008	0.000138	0.002973	0.030008	0.007703	0.966540	0.008279
72	0.000152	0.003638	0.056403	0.058886	0.015669	0.825815	0.052031
73	0.000115	0.003438	0.185056	0.007139	0.000826	0.003937	0.786999
74	0.000092	0.002529	0.108301	0.007483	0.000858	0.004952	0.863231
75	0.000264	0.005486	0.177689	0.295180	0.090257	0.129943	0.301960
76	0.000300	0.009101	0.171540	0.007387	0.001039	0.005087	0.793065
77	0.000030	0.000850	0.032058	0.002548	0.000281	0.001657	0.949706
78	0.000009	0.000245	0.010167	0.001314	0.000132	0.000890	0.974293
79	0.000059	0.001128	0.035592	0.226003	0.019509	0.586832	0.140039
80	0.000004	0.000076	0.002105	0.032412	0.006600	0.969619	0.004917

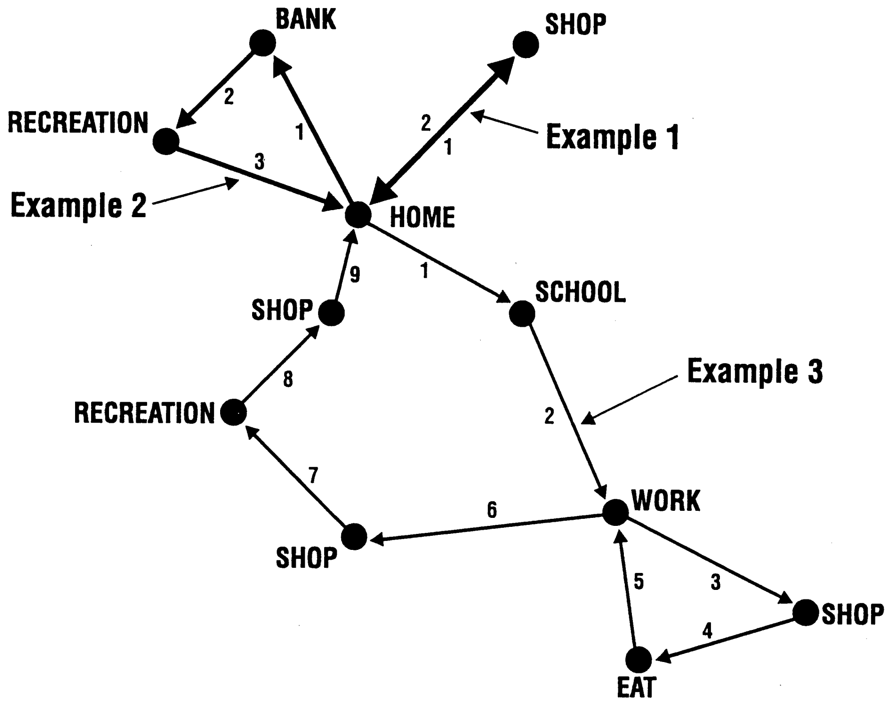
BANK
SHOP
RECREATION
2
1
2
1
Example 1
3
Example 2
HOME
9
1
SHOP
SCHOOL
Example 3
8
RECREATION
2
7
WORK
6
SHOP
5
3
SHOP
4
EAT

Fig 1

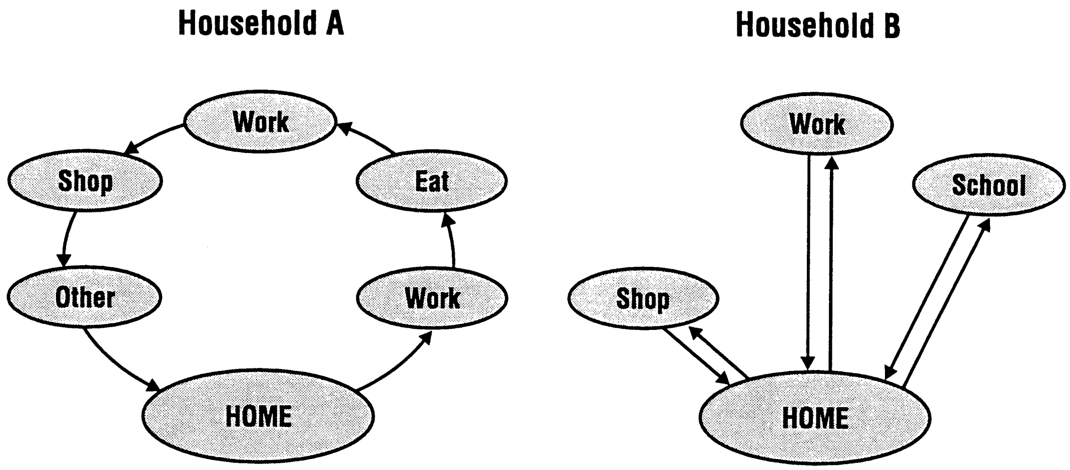
Household A
Work
Shop
Eat
Other
Work
HOME
Household B
Work
School
Shop
HOME

Fig 2

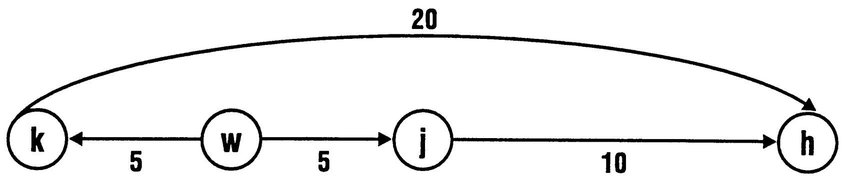

Fig 3