NISS

Some Statistical Properties of Link Travel Times

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Technical Report Number 44
May, 1996

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SOME STATISTICAL PROPERTIES OF LINK TRAVEL TIMES

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Abstract

These notes describe some sources of variability in vehicle travel times when traversing a *link* as part of an urban journey. The aim is to look at possible inter-relationships between these times on adjacent links, and to disentangle sources of variability regardless of the size of that variability, so that some sort of simulation from (conditionally) independent components becomes possible. The approach is a mix of theoretical and empirical. There are some suggestions as to how link travel times might be simulated parametrically.

The data-based analyses are based on measurements made by 'probe' vehicles that traversed a variety of routes in a suburban area of Chicago between 1pm and 7pm on many weekdays in the period June to August 1995. These data are described more fully in other project reports.

Acknowledgements.

The data analysed in this report come from field tests in the northwest suburbs of Chicago made possible by the evaluation of ADVANCE, an Advanced Traveler Information System project. The data were collected and prepared under the supervision of Vonu Thakuriah of the National Institute of Statistical Science, amongst others.

I thank Vonu Thakuriah, Todd Graves, Alan Karr and Jerry Sacks for discussion concerning aspects of these analyses. I am responsible for errors.

Funding for this research was provided in part by NSF-DMS 9313013 to NISS.

1. Components of Link Travel Times: Motivation

It is trite to observe that ideally a link travel time TT can be decomposed as in

$$\mathrm{TT} \approx T_0 + T_{v_{\max}} + T_{\mathrm{intermed}}, \tag{1}$$

where the three terms on the right-hand side denote the durations when a vehicle with travel time TT is respectively stationary, moving at (or about) its 'free flow' speed $v_{\max}$, and moving at any speed intermediate between 0 and $v_{\max}$. This first section discusses our motivation for choosing a particular relation between TT and the other two observed variables CT and CD defined below.

In the study, on each occasion that a vehicle traversed a link there were recorded

TT = time in secs to traverse the link;

CT = time in secs (during TT) that vehicular speed $<$ 2m/sec = 6.6 ft/sec = 4.5mph; and

CD = distance in m. covered (during TT) at speed $<$ 10m/sec = 33ft/sec = 22.5mph.

The time of day was also recorded, together with the same sort of data on any preceding and succeeding links, the totality of which in general constitute a *journey*.

First we relate CT and CD to TT via equation (1). We suppose that $T_0 \approx \mathrm{CT}$, because a vehicle moving at speed $<$ 2m/sec covers a negligeable distance unless the time is exceedingly large. Next, if a vehicle covers a distance D at speeds intermediate between 0 and $v_{\max}$ and the link is of length LD then

$$T_{v_{\max}} \approx \frac{\mathrm{LD} - D}{v_{\max}}; \tag{2}$$

the particular values of the variables on the right-hand side do not concern us for the moment. Substituting these identifications in (1) and rearranging gives

$$\mathrm{TT} \approx \frac{\mathrm{LD}}{v_{\max}} + \mathrm{CT} + \left(T_{\mathrm{intermed}} - \frac{D}{v_{\max}}\right). \tag{3}$$

For the last term here, suppose first that T_{intermed} arises from a single period when the vehicle accelerates from rest to $v_{\max}$ under uniform acceleration f. Then (cf. (2)) the vehicle covers a distance D in this time, and from elementary mechanics $D = \frac{1}{2}(0 + v_{\max})T_{\mathrm{intermed}}$, equivalently $T_{\mathrm{intermed}} = 2D/v_{\max}$; thus the last term in (3) simplifies to $D/v_{\max}$. Similarly, the time t_{crit} taken to reach the speed $v_{\mathrm{crit}} \equiv 10\mathrm{m/sec}$ satisfies $\mathrm{CD} = \frac{1}{2}(0 + v_{\mathrm{crit}})t_{\mathrm{crit}} = \frac{1}{2}v_{\mathrm{crit}}^2/f$, since $v_{\mathrm{crit}} = f t_{\mathrm{crit}}$. Similarly $D = \frac{1}{2}v_{\max}^2/f$, so $D = (v_{\max}/v_{\mathrm{crit}})^2\mathrm{CD}$. Thus, if CD arises from a single period of length T_{intermed} while the vehicle accelerates uniformly from rest to $v_{\max}$, then

$$T_{\mathrm{intermed}} - \frac{D}{v_{\max}} \approx \frac{(v_{\max}/v_{\mathrm{crit}})\mathrm{CD}}{v_{\mathrm{crit}}}. \tag{4}$$

Observe that this relation is independent of the acceleration f which could equally well be deceleration. Relation (4) also holds if CD (or, in (2), the distance D) arises as an accumulation over several periods of acceleration and/or deceleration. In other words, if a vehicle spends all its time either travelling at $v_{\max}$, or being at rest, or uniformly accelerating or decelerating from 0 to $v_{\max}$ or the reverse, then relation (4) still holds. Under these broader assumptions leading to (4), relation (3) yields

$$\mathrm{TT} \approx \frac{\mathrm{LD}}{v_{\max}} + \mathrm{CT} + \frac{(v_{\max}/v_{\mathrm{crit}})\mathrm{CD}}{v_{\mathrm{crit}}}. \tag{5}$$

Consider an alternative scenario under which the vehicular motion occurs at constant power. If the motion starts from rest as above then its speed v at time t later satisfies $v^2 = Ct$ for a constant C. The displacement x at any time t satisfies $dx/dt = \sqrt{Ct}$ so $x = \frac{2}{3}t\sqrt{Ct} = \frac{2}{3}vt = \frac{2}{3}v^3/C$. Under this scenario the last term of (3) evaluates to $\frac{1}{2}D/v_{\max}$, and $D/\mathrm{CD} = (v_{\max}/v_{\mathrm{crit}})^3$, so in place of (4) we should have

$$T_{\mathrm{intermed}} - \frac{D}{v_{\max}} \approx \frac{(v_{\max}/v_{\mathrm{crit}})^2\mathrm{CD}}{2v_{\mathrm{crit}}}, \tag{4$'$}$$

and for (5),

$$\mathrm{TT} \approx \frac{\mathrm{LD}}{v_{\max}} + \mathrm{CT} + \frac{(v_{\max}/v_{\mathrm{crit}})^2\mathrm{CD}}{2v_{\mathrm{crit}}}. \tag{5$'$}$$

Note that (4′) and (5′) hold under the broader assumptions for (4).

The constant on the right hand side of (4) can be either decreased or increased when 'edge effects' are considered, such as a scenario where a vehicle starts its motion on the interval of length LD at positive speed, accelerating towards $v_{\max}$. See the Appendix for detail.

It is arguable that relation (5) should be corrected by a 'start-up' constant depending on whether CT $>$ or $=$ 0, i.e. that one should look for a relation of the type

$$\mathrm{TT} \approx \frac{\mathrm{LD}}{v_{\max}} + \mathrm{CT} + (\text{const.})\mathrm{CD} + (\text{const.})I_{\{\mathrm{CT}>0\}}. \tag{6}$$

All this prompts the investigation of the various regression relations below:

$$\begin{array}{lr} \text{(a)} & \mathrm{TT} = a + b\,\mathrm{CT} + c\,\mathrm{CD} + d\,I_{\{\mathrm{CT}=0\}} + \text{error}; \\ \text{(b)} & \mathrm{TT} - \mathrm{CT} = a + c\,\mathrm{CD} + d\,I_{\{\mathrm{CT}=0\}} + \text{error}; \\ \text{(c)} & \mathrm{TT} = a + b\,\mathrm{CT} + c\,\mathrm{CD} + \text{error}; \\ \text{(d)} & \mathrm{TT} - \mathrm{CT} = a + c\,\mathrm{CD} + \text{error}. \end{array} \tag{7}$$

2. Analyses of Link Travel Time Data

2.1. Regression Analyses using Models (a)–(d)

The *measurements* that underlie the core of this note and that we have used to study these and other models, involve travel times on one of three related links, called *study links* for short. All three start at the same point immediately downstream from a certain signalized intersection; they end at one of three corresponding points at the next signalized intersection after the vehicle has, respectively, proceeded Through or made a Right or Left turn at the latter intersection. See the map at Figure 1 in which the legend identifies certain journeys used in Section 4.

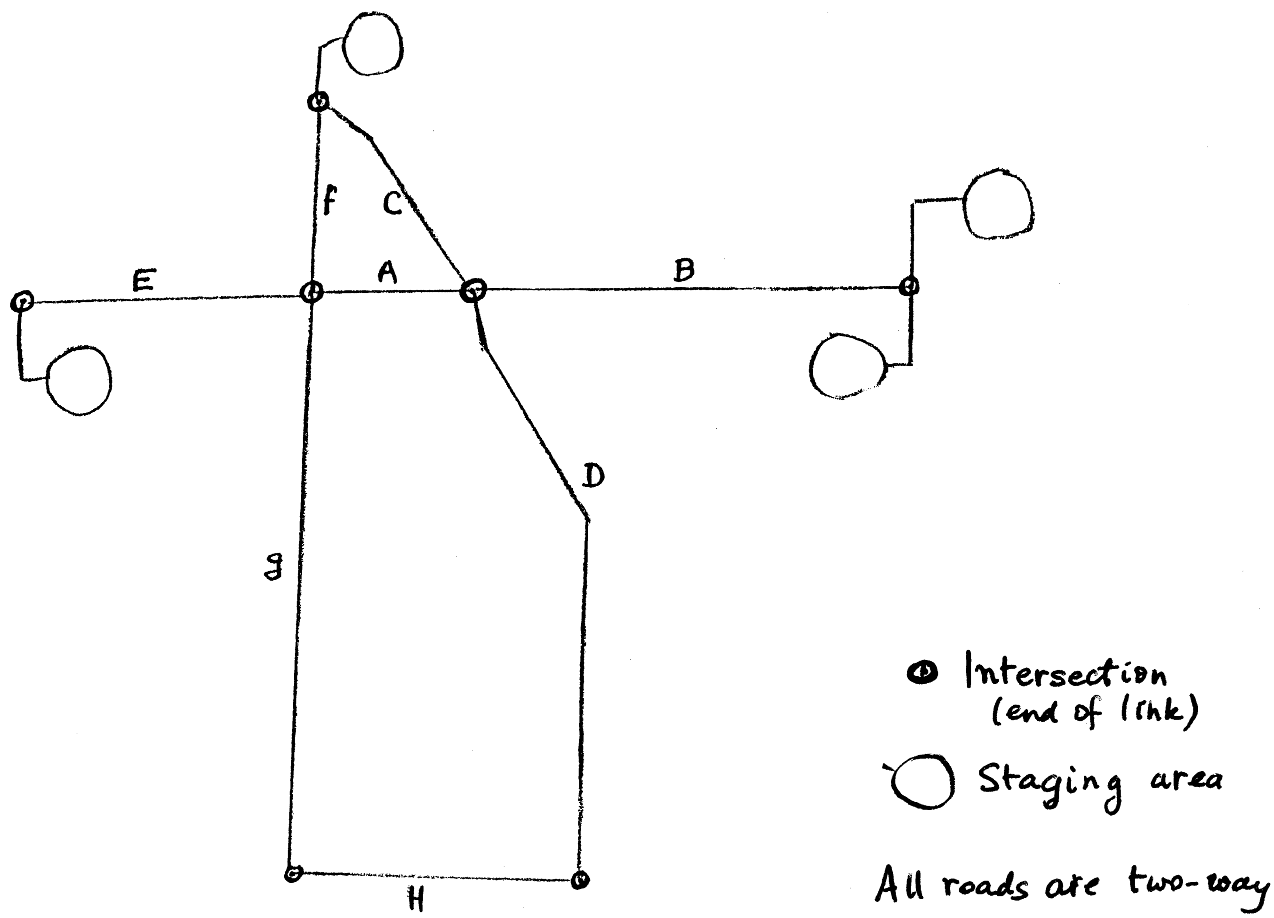

Figure 1—*Map of study link area (approximately to scale).*

Study links: 'Through' = A (to E) ≡ AE ; 'Right' = Af ; 'Left' = Ag

Journeys:	ThTh	ThR	ThL	RTh	RR	RL	LTh	LR	LL
	BAE	BAf	BA(via g)H	CAE	CAf	CA(g)H	DAE	DAf	DA(g)H

The results of some regression studies are shown in Table 1. They give e.g. for model (a) for Tuesday August 1, on the Through link,

$$\mathrm{TT} = 29.53 + 0.822\,\mathrm{CT} + 0.0843\,\mathrm{CD} + 7.172 I_{\{\mathrm{CT}>0\}} \pm 5.16,$$

where $\pm\,5.16$ denotes a residual s.d. of this size. In general, all regression coefficients of CD are highly significantly different from zero; the regression coefficients of CT are significantly different from 1 only when both the correction factor $I_{\{\mathrm{CT}=0\}}$ is included and its coefficient differs significantly from 0, indicating that the regression relation changes according to whether CT = or > 0. In turn this may reflect some weak dependence that exists between CT and CD (cf. Table 8 below).

The length of the Through link is 427 m.; thus the Right- and Left-turn vehicles, which negotiate smaller and larger distances through the intersection at the downstream end of the link, have LD ≈ 417 m and 437 m respectively. Then taking $v_{\max} \approx \mathrm{LD}/\mathrm{const.} \approx 427/29.4 = 14.5\,\mathrm{m/sec}$ $= 32.5\,\mathrm{mph}$, we should expect from (4) that the coefficient of CD would be about $(14.5/10)/10 =$

TABLE 1

Regression coefficients for various models and datasets

Dataset	const	CT	CD	$I_{\{CT>0\}}$	s.d.(error)
	Monday Jul 24, data as at 27 July				
Through	29.27	0.854	0.1001	4.929	5.73
$N = 90$	29.08	1	0.1051	[0.015]	6.23
	29.93	0.910	0.1126		5.91
	29.08	1	0.1052		6.19
Right	28.08	1.013	0.0809	[1.086]	4.60
$N = 87$	28.05	1	0.0812	[1.383]	4.57
	28.29	1.031	0.0825		4.59
	28.42	1	0.0851		4.59
Left	25.40	0.930	0.1168	2.043	5.86
$N = 81$	25.74	1	0.1131	[0.280]	5.99
	26.63	0.943	0.1183		5.86
	25.93	1	0.1135		5.95
	Monday July 31				
Through	28.37	0.777	0.1119	5.765	5.57
$N = 101$	28.30	1	0.1148	[−1.215]	6.18
	28.69	0.859	0.1281		5.74
	28.17	1	0.1090		6.16
Right	27.43	0.868	0.0944	5.341	5.64
$N = 92$	27.19	1	0.0978	[0.806]	6.00
	27.22	0.935	0.1098		5.85
	27.16	1	0.1015		5.98
Left	22.83	0.910	0.1403	1.403	6.41
$N = 75$	23.31	1	0.1352	[−0.507]	6.50
	23.77	0.921	0.1407		6.38
	22.93	1	0.1348		6.48
	Tuesday Aug 01				
Through	29.53	0.822	0.0843	7.172	5.16
$N = 110$	29.56	1	0.0832	[0.659]	5.81
	30.58	0.914	0.0980		5.53
	29.74	1	0.0857		5.79
Right	27.82	0.904	0.0796	3.941	4.98
$N = 105$	27.91	1	0.0748	1.545	5.13
	27.95	0.963	0.0889		5.13
	27.95	1	0.0847		5.14
Left	25.16	0.894	0.1119	4.207	5.61
$N = 96$	26.02	1	0.1032	1.623	5.86
	27.37	0.938	0.1130		5.78
	26.98	1	0.1057		5.87
	Friday Aug 04				
Through	29.83	0.857	0.0899	4.973	5.11
$N = 92$	29.93	1	0.0870	[0.535]	5.51
	30.56	0.922	0.1001		5.30
	30.05	1	0.0891		5.49
Right	29.43	0.962	0.0702	2.272	3.53
$N = 100$	29.43	1	0.0701	[1.256]	3.55
	29.53	0.999	0.0747		3.59
	29.53	1	0.0746		3.58
Left	26.81	0.895	0.1091	1.954	6.56
$N = 89$	27.39	1	0.1025	[−0.744]	6.87
	27.91	0.908	0.1108		6.56
	26.93	1	0.1014		6.84
Approx s.d.	1.0	0.04	0.01	1.5	

Coeffs. 'N.S.' in [. .]

0.145, which is outside the range of values 0.07 to 0.14 in Table 1.[1] The coefficient from relation (4′) evaluates to $(1.45)^2/2(10) = 0.105$, which lies within the observed range of values.

2.2. Travel Times for which CT = 0

About half of the observed TT on Through and Right links have CT = 0. Such CT arise from vehicles which experience no significant delay, but they can still have non-zero CD values through e.g. slowing down, especially right-turning vehicles. In other words, vehicles with CT = 0 have link travel times that reflect 'free flow' suburban arterial conditions subject to minor fluctuations.

Even more pointedly, significant numbers of Through vehicles with CT = 0 also have CD = 0. For example, the TT-values for August 4 for the Through link for which CT = CD = 0 are as follows:

29, 33, 31, 27, 29, 27, 31, 31, 33, 31, 31, 29, 33, 27, 31, 31, 33, 29, 31, 33, 25, 31, 33, 29, 23.
These TT-values have mean 29.6 sec, s.d. 2.4 sec. Their variation may arise from (a) different entry modes onto the study link, (b) different driver characteristics, (c) different 'time of day' effects, and (d) other 'intrinsic variability' due to minor slowing from $v_{\max}$ but insufficient to reach v_{crit}. Subsequent analysis of a dataset with four days' data pooled showed that link travel times for which both CT and CD are zero, have TT-values and frequencies as below:

TT-value:	25	26	27	28	29	30	31	32	33	34	35
Frequency:	5	4	16	5	19	1	13	2	15	0	5

These 85 observations have mean 29.64 and s.d. 2.76. That most of these TT-values are odd integers is an unexplained quirk of the data. Subject to this quirk, the data appear to be approximately uniformly distributed on the interval (25.0, 34.6) (see Figure 2); subsequent plots by Todd Graves using similar data from all days' observations indicate that a normal distribution fits the data adequately, subject to acceptance of the quirk as noted.

The variability in TT-values when CT = 0 can be interpreted as intrinsic variation of TT, including variability between drivers. In particular, the unexplained variance in the regression fit of $\text{TT} = a + c\,\text{CD}$ within the subset CT = 0 yields an estimate of this *noise* or *intrinsic variability* of link travel times. Some fits are shown in Table 2. Observe that the estimates of a are generally in line with those of Table 1 for the same day, whereas the estimates for c tend to be lower, indicating on-average faster travel, as is consistent with CT = 0. The range 0.055 to 0.09 observed for c in Table 2 now excludes the value 0.105 consistent with equation (4′) noted above.

2.3. Congestion Distance CD

In developing the regression relations at (7) we assumed around (4) that CD arises from accelerating or decelerating between speeds 0 and $v_{\max}$. CD may also include distance travelled

[1] Reading from a street-map before the distance 427 m was advised, we estimated LD ≈ 460 m for Through link vehicles. This gives 15.6 m/sec in place of 14.5, and the resulting estimates for (4) and (4′) are 0.156 and 0.122.

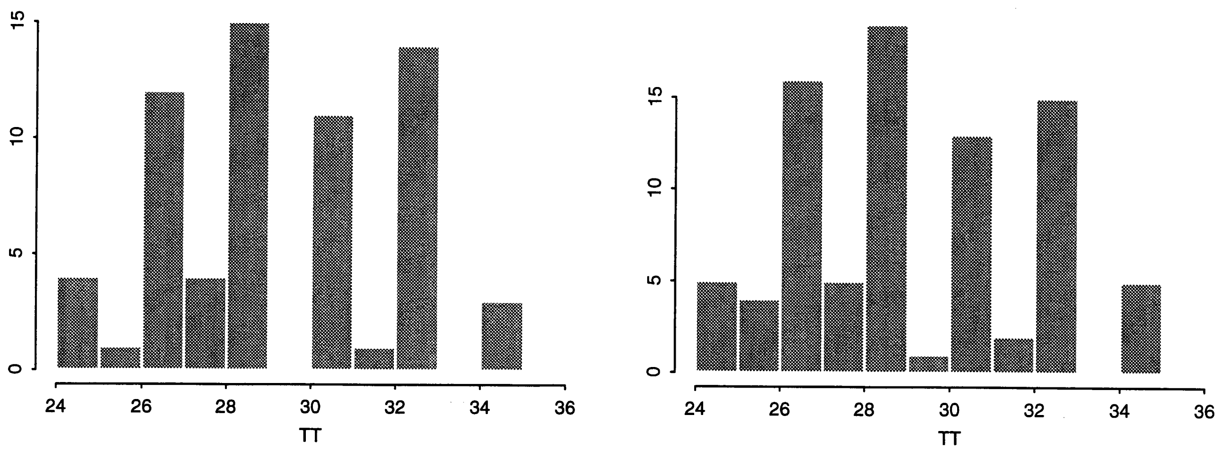

Figure 2—*Histograms of* TT-*values on study link with* CT = 0 *and* CD = 0 *for Journey* ThTh *(left) and All journeys (right).*

TABLE 2

Regression coefficients for data with CT = 0

Dataset	const	CD	s.d.(error)
Monday July 31			
Through ($N = 52$)	28.88	0.0924	2.77
Right ($N = 52$)	29.16	0.0699	2.56
Tuesday Aug 01			
Through ($N = 41$)	30.04	0.0690	2.47
Right ($N = 53$)	29.69	0.0555	2.74
Friday Aug 04			
Through ($N = 39$)	30.77	0.0614	3.17
Right ($N = 48$)	29.27	0.0719	2.48

while temporarily slowed from $v_{\max}$, as for example local transient 'partial' congestion. By way of illustration refer to the scatter plots in Figure 3 of link travel times TT and congestion distance CD for vehicles traversing either the Through or Right-turn link and having zero congestion time (i.e. CT = 0, and for turning vehicles, the turn is executed at speed > 2m/sec).

From the regression analyses it emerges that the distinction between CT = 0 and CT > 0 is of some significance in understanding the variability properties of TT.

It suggested to us that CD should be interpreted as having two components, one for 'slower travel', as when CT = 0, and the other to include acceleration and deceleration between the limits 0 and $v_{\max}$.

A feature of the regression fits in Table 1 is that the constant for Left-turn vehicles is significantly smaller than those for the Through or Right-turn vehicles, at the same time as the coefficient of CD increases (according to another, cruder, argument underlying (4), this would be consistent with the distance CD being traversed at a slower mean speed for Left-turn vehicles than the others).

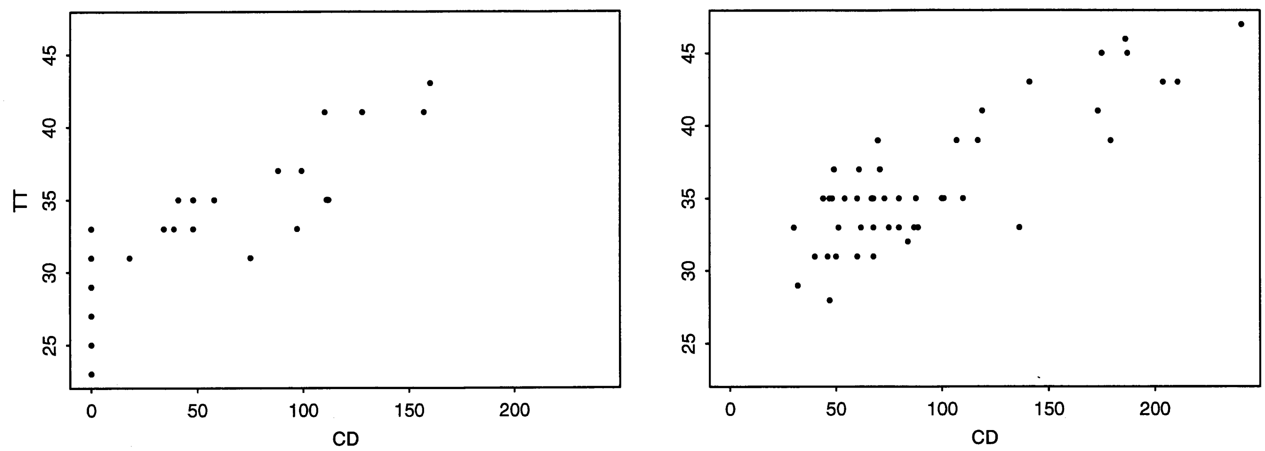

Figure 3—*Scatter plots of* TT *and* CD *for which* CT = 0, *Aug 4 data, for Through link (left) and Right-turn link (right).*

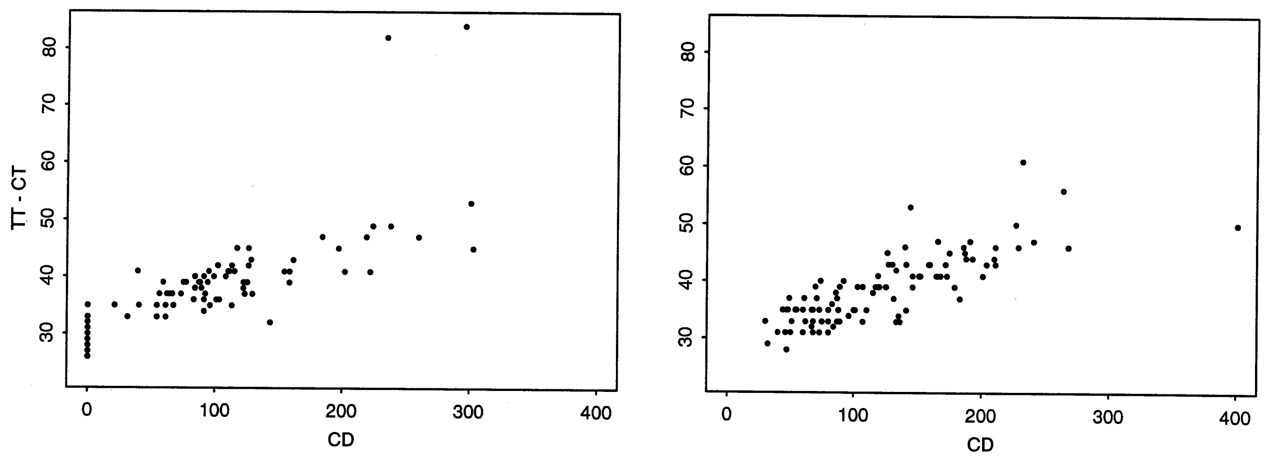

Figure 4—*Scatter-plots of* TT − CT *v.* CD, *Aug 4 data, on Through link (left) and Right-turn link (right).*

2.4. Scatter Plots and Histograms

Inspection of scatter plots in Figure 4 of TT − CT *v.* CD on Through and Right-turn links in Figure 4 suggests that, subject to the exclusion of outlying observations, TT − CT lies in a band about 10 to 12 sec wide, as is consistent with an intrinsic variability component of TT with a s.d. about 2.7 sec.

For Right-turn vehicles, the component of CD due to reducing speed below 10 m/sec to execute the right-turn movement is in the region 20 to 100 m.

Figure A.1 shows various plots of CT and CD after pooling four days' worth of data. The scatter plot suggests some sort of positive correlation, though not strong. The histogram of CT-values (lower-left) reveals that about 40% of these values are zero, while the rest are not quite uniformly distributed on the interval (0,60). In view of the concentration of values of CT at 0, the CD-values are presented in two histograms, one for CT = 0 and the other for CT > 0. Immediately a difference in the CD-values is revealed: about 20% of the observed CT = 0 have CD = 0 (since acceleration is finite, it is not possible to have CD = 0 and CT > 0). Further, values of CD conditional on both being positive and CT = 0, are on average smaller than values for which CT > 0.

A further disaggregation of both CT and CD values according to the movements at both the beginning and end of the study link reveals more, as we show in Section 4.

2.5. 'QQnorm' Plots

In plots (not shown) of the ordered regression residuals (of actual − fitted values) against ordered normally distributed residuals with same standard deviation at the same quantile, the data show moderate agreement (as evidenced by linearity) for all except the upper tail, i.e. there are typically up to (say) 5 or 10% residuals for which the actual value is much larger than the fitted, and larger than would be expected of normally distributed residuals. What this says is that deviations from a regression relation are not normally distributed: they are heavier tailed in the positive direction. And this is 'predictable': travel times involve only excessive delays rather unduly speedy trips!

2.6. Vehicles

Table 3 presents analyses of crude data on vehicles for August 4. The same linear model was fitted to data on all links travelled (a) by all vehicles, and (b) by each of the 'vehicles' identified via the modem fitted in it. Note that *all* links are used in this study. The major contributory factor to the considerably larger residual s.d. in Table 3 is the diversity of links included (hence, of mean link travel times, i.e. of the constant terms in the link TT regression relations), and travelled by each vehicle. There is no other significance in the size of these values.

Note that in the design of the study, routes were randomly allocated to vehicles, subject to having a reasonably high frequency of journeys that included the study link.

TABLE 3

Regression coefficients for link travel times by vehicle

(Modem)	N	const	CT	CD	s.d.(error)
		Friday Aug 04			
All	1051	40.33	0.967	0.1134	27.93
	(s.e.s:	1.37	0.029	0.0102)	
16	145	46.00	0.850	0.1395	31.96
	(s.e.s	5.35	0.113	0.0400)	
19	141	38.79	0.995	0.1107	25.91
20	134	49.41	0.878	0.0881	30.92
25	145	40.38	0.901	0.1125	27.27
62	159	37.31	1.085	0.0765	23.03
66	114	36.86	1.254	0.0630	28.61
69	148	41.35	0.965	0.1004	26.88
80	122	33.49	1.017	0.1525	28.83
89	156	42.08	0.858	0.1230	27.21
92	132	40.13	0.962	0.1145	29.30

3. Regression Fits: Interpretation

Travel times and their variability can be described via components as below:

(1) T_{noise} comprises small 'noise' or intrinsic variability of TT with no significant congestion effects (marginally altered traffic conditions, variation between drivers): s.d. about 2.7 sec.

(2) T_{signal} comprises delay due to one traffic-light stop; under conditions of light-to-moderate traffic flow, if the signals are periodic (rather than vehicle-actuated) over a time d with the red-phase of length $d_r < d$ and vehicle arrival times at the signal are randomly distributed on $(0, d)$, we then have a proportion of about $p_r \equiv d_r/d$ vehicles that experience such a stop which has duration uniformly distributed on $(0, d_r)$ with mean $\frac{1}{2}d_r$ and s.d. $\frac{1}{2}d_r/\sqrt{3}$.

(3) T_{minorinc} arises from 'minor' 'incidents' of short-term transient local effect (e.g. another vehicle joining a traffic stream in a gap not long enough to allow full acceleration to $v_{\max}$, or vehicle slowing on approaching a queue of vehicles moving off at a traffic signal just turned green) that causes a temporary slowing which in grosser circumstances contributes to CD.

(4) $T_{v_{\max}}$ is a 'free-flow' component when the vehicle covers a distance LD $- D$ at speed c. $v_{\max}$.

(5) There may also be exceptional delays due to some unusual happening(s), affecting (say) 5% of all link travel times.

If we were to predict link travel times TT_{pred} omitting cause (5) above, these interpretations would yield

$$\text{TT}_{\text{pred}} = T_{v_{\max}} + T_{\text{signal}} + T_{\text{minorinc}} + T_{\text{noise}}.$$

In a journey comprised of several links, allowance must be made for the exceptional factor (5) as the union of several such events, which we assume to be statistically independent, constitutes an event of probability much larger than around 0.05.

4. Journeys on Connected Links

4.1. Up- and Down-stream Links

There are three links immediately upstream from the study link, and involve traffic entering the study link either directly or via a right or left turn. Similarly, there are three links immediately downstream from the study link; the vehicles enter these subsequent links either directly or after a right or left turn respectively.[2] Table 4 gives regression fits of link travel times for these links; we refer to them subsequently in connection with journey times. Figure A.2 shows plots of these fitted values for the Through link and both the Through and Right-turn links into the study link. These plots are given to illustrate the similarity of the 'noise' component of link travel times (cf. the scatter plots at Figure 4).

[2] The data collected are not quite as simple as this sounds. Specifically, with each of the three study links there is associated exactly one downstream link. For Through and Right turn vehicles, this link is indeed the one immediately downstream. For vehicles turning left at the end of the study link the data relate to the second link downstream; this link is in fact parallel to the study link, and is entered via a left turn at the end of the first link after the study link (link H in the map at Figure 1).

TABLE 4

Regression coefficients for links up- and downstream from study link

Dataset	N	const	CT	CD	s.d.(error)
			Friday Aug 04		
			Upstream links		
Through	147	29.62	0.909	0.0784	7.14
Right	67	42.10	0.972	0.0973	3.78
Left	67	110.80	0.861	0.1693	12.66
			Downstream links		
(Through)	102	45.21	0.980	0.1312	5.57
(Right)	101	37.35	0.978	0.0755	6.07
(Left)	93	21.15	0.933	0.1770	4.98

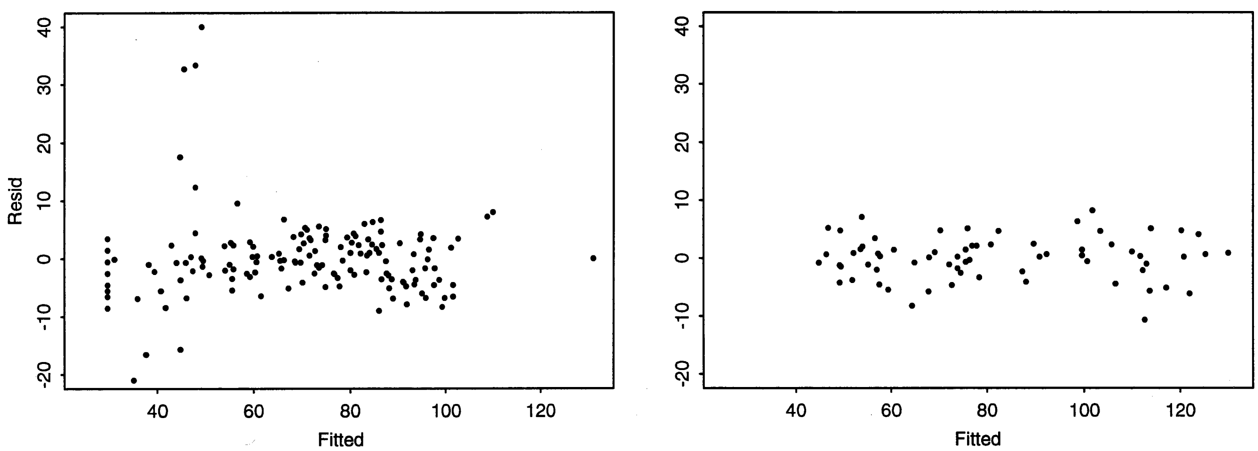

Figure 5—*Scatter-plots of residual v. fitted values from regression model (c) for* TT *on Through (left) and Right-turn (right) links upstream from study links.*

Figure 5 shows plots of these fitted values for the Through and Right-turn links into the study link. These plots are given to illustrate the similarity of the 'noise' component of link travel times, whether the study link or otherwise. We note that on the upstream link connecting through the intersection onto the study link, some actual TT-values are much larger than the regression equation predicts: we have not studied this matter.

4.2. Journey Times

Table 5 lists the results of regression fits of model (c) for 9 different 'journeys' each consisting of three links, the central one being one of the three study links, the first one of the three links immediately prior to one of the study links, and the last the downstream link (this was always the same for a given study link).

Table 6 presents further summary data concerning these journey travel times and congestion measures CT and CD. For each of the 9 journeys, the three lines represent moments of data relating to[3] $\mathrm{TT}_{\mathrm{std}} \equiv \mathrm{TT} - \mathrm{CT} - 0.12\,\mathrm{CD}$, CT and CD, on each of the three links and on the journey (=

[3] In the first computations we did for this note, as in the bulk of Tables 6 and 7, we used the coefficient

TABLE 5

Regression coefficients for journey times

Journey	N	const	CT	CD	s.d.(error)
ThTh	32	103.42	0.885	0.1095	9.56
ThR	57	93.50	0.992	0.0756	10.77
ThL	47	80.66	1.093	0.0866	23.01
RTh	23	110.49	0.948	0.1283	12.72
RR	17	114.73	0.923	0.0798	5.82
RL	21	102.35	0.868	0.1074	10.73
LTh	24	189.00	0.827	0.0695	13.14
LR	22	184.46	0.897	0.1017	12.70
LL	18	125.00	1.036	0.2190	12.45

all three links), while s.d.(sumVar) $= \sqrt{\sum_{j=1}^{3} \mathrm{var}_{\mathrm{Link}_j}}$. Thus, if the components on the links are uncorrelated then s.d.(sumVar) $\approx$ s.d.(Jny). This is true of $\mathrm{TT}_{\mathrm{std}}$, and 'true' within slightly wider bounds of CT, but for CD there is the suggestion that s.d.(Jny) $>$ s.d.(sumVar), indicating weak positive correlation between distances covered on consecutive links at speeds $< v_{\mathrm{crit}}$, such as we should expect to arise from

- turning movements or traffic light effects at contiguous ends of links;
- time of day effects;
- drivers' different reactions to congestion and practices in braking and accelerating.

The summary data in Table 6 enable us to disentangle the effects of movement at the upstream end of the study links on each of the three measures relating to travel on each of the three study links. For example, using the nine $\mathrm{TT}_{\mathrm{std}}$ averages $28.30, \ldots, 24.07$ there, the mean $\mathrm{TT}_{\mathrm{std}}$ can be calculated (a) for a given type of exit movement from the study link A, and (b) for a given type of exit movement from the upstream link immediately prior to entry onto A. The results are shown in Table 7. There are discernible effects on all of TT, CT and CD, and what is measured in CT and CD is largely removed from TT in constructing $\mathrm{TT}_{\mathrm{std}}$ in the sense that these standardised travel times are relatively little different. The congestion times and congestion distances both show effects that one would anticipate. With regard to the nature of the exit from the study link there is

(i) decreased CT on a right-turn movement relative to through movement due to either or both of a smaller proportion of vehicles turning right (hence, smaller queue of vehicle forms when light is red), and right-turn allowed after stop on red;

(ii) increased CT on a left-turn movement, due to smaller phase of cycle when left-turn is protected against oncoming traffic;

(iii) increased CD, perhaps moreso for left-turn vehicles, due to the need to slow before exiting the length of the link; mean distance for slowing is between 40 and 60 m.

0.12 for CD in $\mathrm{TT}_{\mathrm{std}}$. Later, notably the last part of Table 6 and Table 8, we used the coefficient 0.10. Comparison of the upper and lower parts of Table 6 shows that the coefficient 0.10 yields smaller var $\mathrm{TT}_{\mathrm{std}}$ for 19 of the 27 independent link-based datasets concerned.

TABLE 6

First and second moments of link and journey measurements

Jny N		Averages Link 1	Link 2	Link 3	Journey	Standard Deviations Link 1	Link 2	Link 3	Journey	sumVar
ThTh	TT_{std}	20.65	28.30	45.08	94.02	8.63	5.22	3.55	10.52	10.69
32	CT	28.22	10.44	14.94	53.59	19.40	22.46	14.38	35.62	32.98
	CD	159.97	70.19	78.50	308.66	93.53	81.98	25.03	143.20	126.87
ThR	TT_{std}	19.28	24.71	30.22	74.22	8.37	3.88	8.18	12.87	12.33
57	CT	30.23	9.63	32.35	72.21	19.16	19.63	26.53	38.39	38.16
	CD	161.53	103.84	156.07	421.44	87.78	58.46	79.19	161.78	131.89
ThL	TT_{std}	20.72	25.63	28.42	74.77	8.67	8.20	20.47	22.97	23.70
47	CT	33.02	23.85	19.85	76.72	19.42	25.74	20.88	42.36	38.41
	CD	156.09	144.19	89.98	390.26	84.01	87.08	32.21	141.29	125.22
RTh	TT_{std}	37.24	27.01	44.97	109.23	3.46	7.49	9.22	12.20	12.37
23	CT	23.78	30.30	17.17	71.26	21.58	21.99	18.79	25.26	36.09
	CD	99.78	101.70	92.26	293.74	47.73	65.24	44.90	105.39	92.47
RR	TT_{std}	40.44	21.94	30.37	92.75	3.63	6.99	5.57	9.93	9.65
17	CT	33.12	19.29	29.41	81.82	25.08	19.44	24.66	37.56	40.19
	CD	101.24	146.59	142.47	390.29	45.81	78.35	81.58	159.04	122.03
RL	TT_{std}	39.17	20.93	27.13	87.23	4.62	5.90	8.08	11.99	11.02
21	CT	34.29	25.33	19.48	79.10	26.28	19.44	15.88	41.35	36.34
	CD	116.81	163.67	90.62	371.10	48.87	79.64	48.89	120.29	105.45
LTh	TT_{std}	112.68	26.42	46.97	186.06	14.24	6.27	3.89	14.65	16.04
24	CT	33.67	19.13	14.54	67.33	29.45	18.46	13.40	39.46	37.25
	CD	155.46	107.29	91.96	354.71	88.97	90.65	34.61	147.95	131.64
LR	TT_{std}	110.90	24.53	28.90	164.32	10.42	3.45	9.00	14.60	14.19
22	CT	40.77	20.86	52.18	113.82	37.98	16.81	26.28	53.39	49.15
	CD	139.50	148.64	168.64	456.77	92.22	57.22	111.03	187.26	155.26
LL	TT_{std}	117.17	24.07	24.92	166.16	18.34	3.19	3.54	19.23	18.95
18	CT	32.17	27.72	15.50	75.39	29.92	23.30	15.48	38.84	40.96
	CD	176.39	144.78	66.89	388.06	138.99	45.32	27.92	155.83	148.83
Jny	N	Moments of $TT_{std} = TT - CT - 0.10\,CD$								
ThTh	32	23.85	29.70	46.65	100.20	7.77	5.28	3.50	9.93	10.02
ThR	57	22.51	26.79	33.34	82.65	7.46	3.12	7.64	11.35	11.12
ThL	47	23.84	28.52	30.21	82.57	8.15	8.49	20.59	22.78	23.72
RTh	23	39.24	29.05	46.82	115.10	3.33	7.48	9.60	12.44	12.62
RR	17	42.46	24.87	33.22	100.56	3.26	5.97	4.35	7.63	8.08
RL	21	41.51	24.20	28.94	94.65	4.39	4.51	8.78	11.38	10.80
LTh	24	115.79	28.56	48.80	193.15	15.03	4.82	3.55	15.58	16.18
LR	22	113.69	27.50	32.27	173.46	9.54	3.32	7.51	13.20	12.59
LL	18	120.69	26.97	26.26	173.92	20.32	2.57	3.67	21.78	20.81

With regard to the nature of the upstream link,

(iv) a Through connnection on entry rather than entry after turning makes for relatively smaller CT and CD, with little difference whether right- or left-turn into the study link. The average distance required for acceleration is about 30 m. from the measurement point at the start of link. (Another possible cause is associated with the phase of the red–green cycle at the intersection near the end of the study link).

In other words, this analysis (of August 4 data, and replicated on data for other days) indicates that including the downstream intersection movement alone in the link descriptor does not explain all the distinct variation in mean link travel times, congestion times CT and distances CD; the upstream movement is still relevant in explaining more of that variation.

TABLE 7

Averages of Study Link Measures by Link type

Measure	Study link type Through	Right	Left	Upstream link type Through	Right	Left
TT − CT − 0.12 CD	27.24	23.73	23.54	26.21	23.29	25.01
TT − CT − 0.10 CD	29.10	26.39	26.56	28.33	26.04	27.68
TT	58.35	56.28	67.28	53.57	64.74	63.61
CT	19.94	16.59	25.63	14.64	24.97	22.57
CD	93.06	133.02	150.88	106.07	137.32	133.57

E.g. (cf. Table 5) $27.24 = \frac{1}{3}(28.30 + 27.01 + 26.42)$, $26.21 = \frac{1}{3}(28.30 + 24.71 + 25.63)$

If we were to associate the variation in CT as arising solely from arrival at the intersection near the end of the study link at a random phase of the traffic light cycle there, then the model associated with T_{signal} in the interpretation of regression fits would yield, for this r.v., mean $\frac{1}{2}p_r d_r$ and s.d. $\sqrt{p_r(4-3p_r)/12}\, d_r$, hence a coefficient of variation equal to $\sqrt{(4/3p_r)-1}$, ≈ 1.3 for $p_r = \frac{1}{2}$. A single day's data is insufficient to determine the adequacy of this model.

Table 8 shows the correlations between the measures $\text{TT}_{\text{std}} = \text{TT} - \text{CT} - 0.1\,\text{CD}$, CT and CD, on each of the three links and over the various 3-link journeys. Recall from Table 1 that the regression coefficients of CT we found for model (c) are mostly less than 1, although not by much. Then the correlation coefficient $r(\text{TT}_{\text{std}}, \text{CT})$ should in general be negative but small. Table 8 shows that the correlation coefficients $r(\text{CT}, \text{CD})$ tend to be positive: it is not altogether surprising that these two measures should be positively correlated, since any congestion time is almost always associated with travelling some distance while accelerating or slowing between travel below 2 m/sec and above 10 m/sec.

TABLE 8

Correlations between Link Measures on Links and Journeys

	$r(\text{TT}_{\text{std}}, \text{CT})$				$r(\text{TT}_{\text{std}}, \text{CD})$				$r(\text{CT}, \text{CD})$			
Jny	Lk1	Lk2	Lk3	Jny	Lk1	Lk2	Lk3	Jny	Lk1	Lk2	Lk3	Jny
ThTh	-.387	-.348	.126	-.345	-.364	.188	-.022	-.067	.533	.629	.086	.494
ThR	-.244	.003	-.244	-.192	-.433	-.547	-.250	-.361	.395	.350	.270	.475
ThL	-.334	-.284	.659	.136	-.213	.265	.201	-.006	.393	.427	.121	.447
RTh	-.022	-.236	-.347	-.015	.004	.086	.455	.200	-.021	.131	-.026	.377
RR	.026	-.390	-.408	-.604	-.284	-.572	-.662	-.623	.347	.456	.193	.535
RL	-.570	-.281	-.371	-.443	-.135	-.830	.749	-.151	.393	.191	-.027	.478
LTh	-.457	-.371	.297	-.416	.490	-.728	-.420	.400	-.283	.538	.050	.051
LR	-.494	-.222	-.201	-.402	-.399	.056	-.590	-.251	.673	-.177	.509	.658
LL	-.212	-.442	.008	-.075	.749	-.599	.302	.841	-.327	.392	.248	-.164

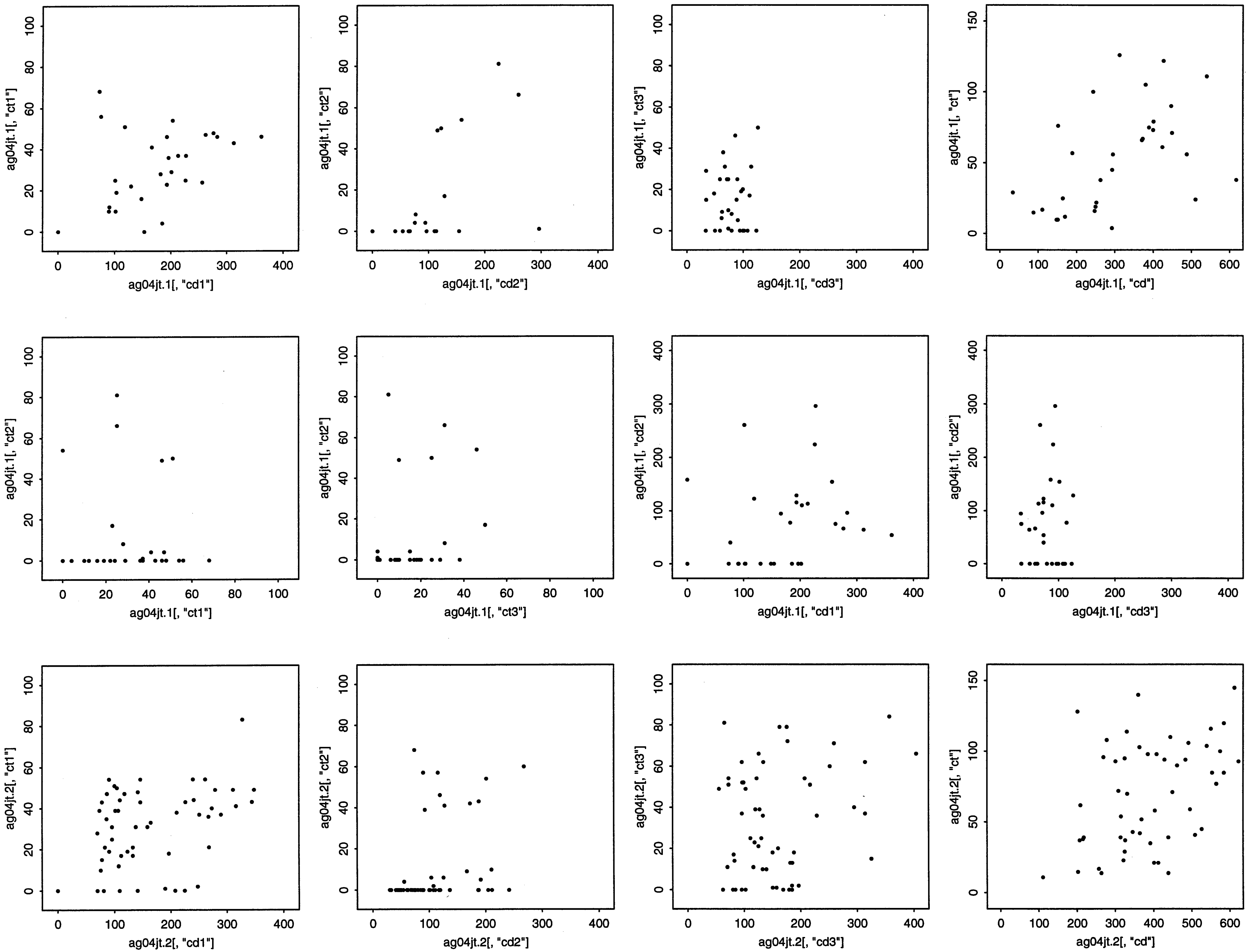
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Figure 6

4.3. Scatter Plots of Link and Journey CTs and CDs

Figure 6 opposite shows scatter plots of the CT and CD values for the three links and the total journey for August 4 on Journey ThTh = BAE (top four boxes) and Journey ThR = BAf (bottom four boxes), together with plots of CT on Link 2 against CT on links 1 and 3 (left-hand pair of boxes on middle row) and CD on Link 2 against CD on links 1 and 3 (right-hand pair of boxes on middle row). The correlation coefficients of the scatter plots at the right-hand end of the top and bottom rows are 0.494 and 0.475 respectively (Table 8).

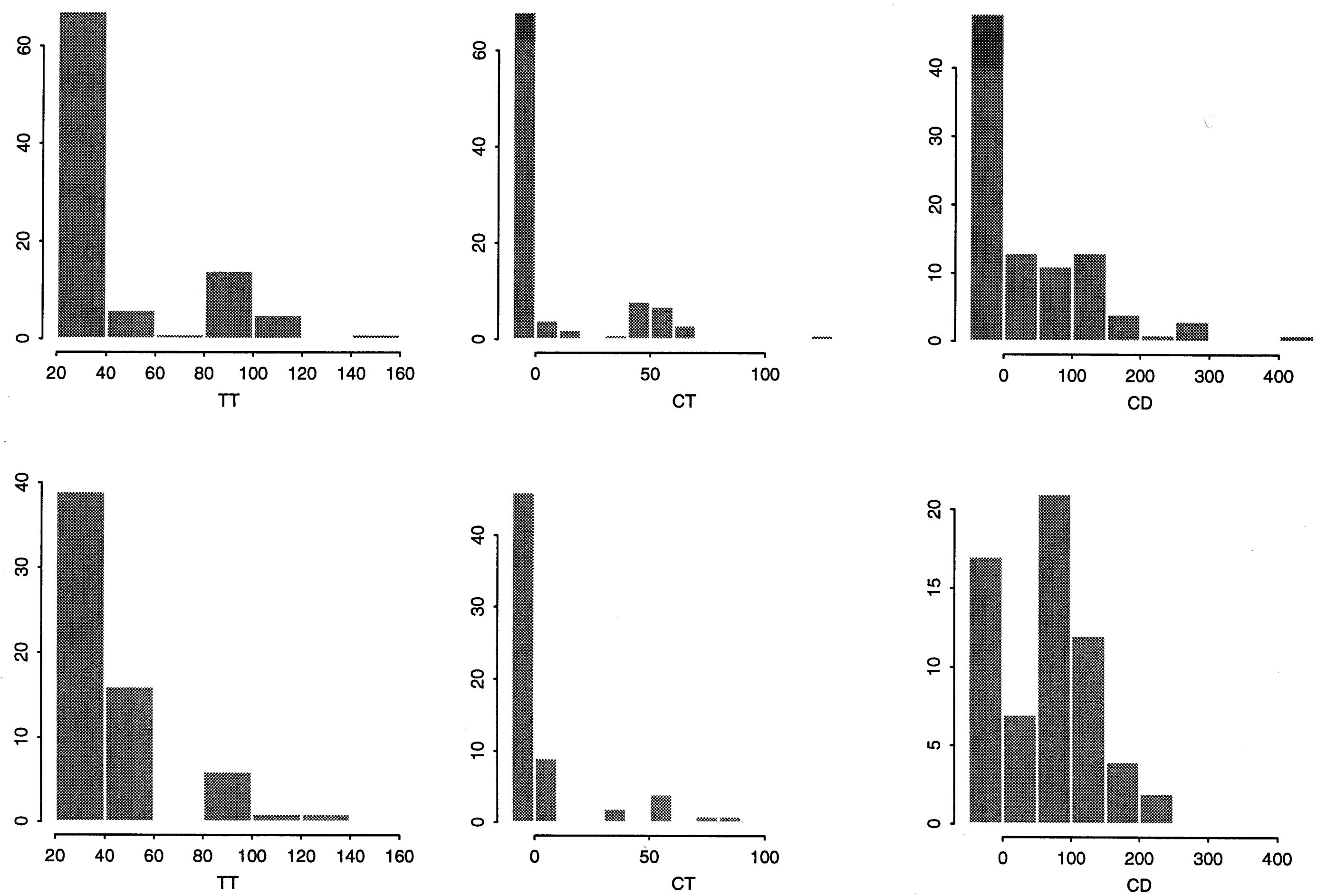

Figure 7—*Histograms of* TT, CT *and* CD *during off-peak (upper row) and peak (lower row) periods*

4.4. Time of day effect

Figure 7 shows histograms of TT, CT and CD for observations made in the period between 4pm and 6pm (roughly speaking, the 'peak period'), in the lower three panels, and in the rest of the observation period in the upper three panels, for vehicles on the Through study link entering it via a Through upstream movement. Relative to measurements in the off-peak period, in the peak period there is an evident upward shift in TT-values below 60 secs, not a great change in CT, and

and an upward shift in CD distances, in particular proportionately fewer vehicles with CD $= 0$ (the means for TT, CT and CD are 44.02, 8.05 and 70.4 ($N = 63$, peak) and 46.42, 12.28 and 53.1 ($N = 94$, off-peak)). The (smallish) decrease in mean TT is associated with decreased mean CT and increased T_{intermed} as reflected in increased CD.

On the basis of these histograms, we should now investigate whether the 'time of day' effect we suggested above in connection with Table 6 is sufficient to account for the apparent correlation between CD on the links of the nine journeys studied there (what we noticed in Table 6 is that for CD, s.d.(Jny) $>$ s.d.(sumVar) for all of the 9 journeys listed there; making similar comparisons for three other days we found the same inequality for 24 out of 27 sets of journey data.

Accordingly we analysed the data pooled over the four days, splitting the dataset into two sets according to peak or off-peak period. The inequality holds for 4 out of 9 journeys in the peak period, and for 7 out of 9 journeys in the off-peak period. This suggests that at least part of the apparent positive correlation between link-based CD s on journeys is attributable to a time-of-day effect.

For all 9 journeys, the mean CD during peak period exceeds the mean CD off-peak on the study link; a t-test supports such a statement at a 95% confidence level for 6 of the 9 journeys.

There was no discernible pattern observed in comparing s.d.(Jny) and s.d.(sumVar) for TT_{std} and CT for the 9 journeys, just as for the one day's data analysed in Table 6.

5. Some Modelling Implications

One object of the analysis in this note is to reduce the modelling of link travel times to a context where independent uniform random variables (r.v.s) can be transformed and combined together via appropriate conditional distributions. The comparisons of standard deviations suggests that the TT_{std} and CT components on a given journey can be constructed as independent r.v.s.

The form that this might take includes the following when modelling TT on link 2 (in the terminology of Table 6), i.e. the study link. We have not at this stage pursued all the detail with regard to any decomposition at the upstream link in terms of the phase of the upstream traffic light cycle etc.: such an analysis should be possible when video data are to hand. For the time being there is suggestive information in the various sets of histograms in Figures A.3–5, of CT (A.3) and CD (A.4) on the study link for each of the nine journeys described above, and in A.5 of CD on the Through (left column), Right (centre) and Left (right column) study links entered from the upstream Through link, for trips with CT $= 0$ (top row of A.5), CT > 0 (middle row) and all four days' data (bottom row, duplicating left-most column of A.4). Similar analysis should be done for the other six journeys.

(1) Determine the journey; this fixes the nature of the upstream link.
(2) Take TT_{std} to be (say) N(mean, 3^2) distributed r.v.s, independent of others that follow below; the mean could be made time-of-day (ToD) dependent and journey-type dependent, though for a first approximation it can be fixed at (say) 29.0 sec.

(3) Model CT as a r.v. with an atom at 0 (and the size of this atom depends on both the up- and down-stream links), and otherwise with a density on (say) (0,60) (a need for larger values arises only in peak periods). For another link the range may need to be a function of the maximum period of traffic light cycle for downstream left-turns.

(4) Model CD as a r.v. conditional on CT as just determined:

   .1) if CT $= 0$ then on the Through study link, CD $= 0$ is possible and otherwise (and for the other two study links) exponentially distributed on (say) (30,200), and

   .2) if CT > 0 then CD is the sum of two independent exponentially distributed r.v.s on (30,200).

A major question here concerns the nature of the distributions (and their parameters): both more data and then either fitting or simulation are avenues worth investigating.

The sum of two shifted exponentially distributed r.v.s has the appealing feature that CT $= 0$ implies both a deceleration and an acceleration have occurred. Why exponentials may be useful is simply a reflection of the possibility of there being other random disturbances that result in the vehicle slowing appreciably (and enough to influence CD).

One possibility for investigating independence of link components is to use a bootstrap simulation technique to sample links at random and join them into journey measures, testing the resulting sums against the observed distribution of journey times.

6. Concluding Remarks

We suggested earlier that, subject to the exclusion of 'extraordinary' events, link travel times TT could be described via four components. Our more detailed analyses involving study link behaviour *vis a vis* some upstream behaviour, supports that view subject to modifications. Clearly we must allow for some dependence between the terms T_{signal} on associated links: this is obvious in the context of a suburban road journey where a delay at any given intersection has measurable consequences in travel times on the two links that end and start in the neighbourhood of that intersection, and where traffic lights at successive intersections may have their cycle times coordinated in some fashion.

Certainly, at the very least, a 'journey' decomposed into 'links' has distinguishable elements that correspond to sectors wholly between intersections and others wholly within neighbourhoods of intersections, because motion on a journey that includes some intersections with synchronized signals introduces dependence of travel ttimes on successive links. For the time being we describe

Probe vehicle data. Suppose that a vehicle routinely gathers information on TT, CT and CD whenever it travels, these data being the basis for automated reporting of any 'extraordinary' traffic conditions. Both the regression analyses and the subsequent first and second moment analyses given above indicate that it is enough to identify outliers in CT and CD (and possibly $\text{TT}_{\text{std}} = \text{TT} - \text{CT} - 0.1\,\text{CD}$) and only report these measures; it is worth 'correcting' TT in doing so. The s.d. of any single observation as a representative of observations from many vehicles, is large.

Appendix: Modelling 'edge effects' in CD

Suppose first that a vehicle has speed v_0 at time $t = 0$ and moves with constant acceleration f through an intermediate speed v_1 to speed v_2, taking times $t_j = (v_j - v_0)/f$ to do so. The distances covered in these times satisfy $d_j = \frac{1}{2}(v_0 + v_j)t_j = \frac{1}{2}(v_j^2 - v_0^2)/f$, so

$$d_2 = \frac{v_2^2 - v_0^2}{v_1^2 - v_0^2}\, d_1. \tag{A.1}$$

If then the term $T_{\text{intermed}} - D/v_{\max}$ at (2) arises solely from a vehicle entering a link at speed $v_0 < v_{\text{crit}}$ and accelerating up to speed $v_{\max}$ which is maintained thereafter while it traverses the link, as may happen for example with some vehicles on the Through link, then in place of (4) we should have

$$T_{\text{intermed}} - \frac{D}{v_{\max}} = \frac{2D}{v_{\max} + v_0} - \frac{D}{v_{\max}} = \frac{v_{\max} - v_0}{v_{\max}(v_{\max} + v_0)}\, D = \frac{(v_{\max} - v_0)^2}{v_{\text{crit}}^2 - v_0^2} \cdot \frac{\text{CD}}{v_{\max}}. \tag{A.2}$$

We assert that the coefficient of $\text{CD}/v_{\max}$ here can be either smaller or larger than the coeffcient $v_{\max}^2/v_{\text{crit}}^2$ as at (4) (corresponding to $v_0 = 0$). This can be checked using elementary calculus and the function

$$f(x) \equiv \frac{(1-x)^2}{a^2 - x^2} \qquad \left(a = \frac{v_{\text{crit}}}{v_{\max}} \approx \tfrac{2}{3}, \quad x = \frac{v_0}{v_{\max}}\right).$$

First find that $f'(0) < 0$. Then find that $\inf_{0<x<a} f(x)$ occurs at $x = a^2$, corresponding to $v_0 = (v_{\text{crit}}/v_{\max})v_{\text{crit}}$, the minimum value being $a^{-2} - 1 = (v_{\max}/v_{\text{crit}})^2 - 1$, and $f(x) \to \infty$ for $x \uparrow a$. $f(x) = f(0)$ in $x > 0$ at $x = 2a^2/(1 + a^2)$, corresponding to $v_0 = [1/\frac{1}{2}(a + a^{-1})]v_{\text{crit}}$, $= \frac{12}{13}v_{\text{crit}}$ for $a = \frac{2}{3}$.

Now suppose that the contributions to CD and D occur over several time intervals where the vehicle is fluctuating between $v_{\max}$ and speeds in the range 2 to 10 m/sec, so CD is positive but CT = 0. Except when this lower speed exceeds c. 9 m/sec, the mix of coefficients for CD would be lower than that at (4). On the other hand when the lower speed exceeds about 9 m/sec, the coefficient would exceed that at (4) (and even moreso, provided CD > 0, when this lower speed of some deviation from $v_{\max}$ exceeds v_{crit}).

Under the alternative, constant power, scenario as above (4′), we should have (in the same notation as there and above) that $v^2 - v_0^2 = Ct$, and $x = \frac{2}{3}(v^3 - v_0^3)/C$ so

$$T_{\text{intermed}} = \frac{v_{\max}^2 - v_0^2}{C} = \frac{3(v_{\max}^2 - v_0^2)}{2(v_{\max}^3 - v_0^3)}\, D,$$

and

$$T_{\text{intermed}} - \frac{D}{v_{\max}} = \frac{\frac{1}{2}(v_{\max}^3 - 3v_{\max}v_0^2 + 2v_0^3)}{v_{\max}(v_{\max}^3 - v_0^3)}\, D = \frac{v_{\max}^3 - 3v_{\max}v_0^2 + 2v_0^3}{2(v_{\text{crit}}^3 - v_0^3)} \cdot \frac{\text{CD}}{v_{\max}}.$$

The same qualitative results hold for the coefficients of CD here and at (4′) as for the coefficients at (A.2) and (4). In place of the function $f(\cdot)$ above one has $f_2(x) \equiv \frac{1}{2}(1 - 3x^2 + 2x^3)/(a^3 - x^3)$ $= \frac{1}{2}(1-x)^2(1+2x)/(a^3 - x^3)$. When e.g. $a = \frac{2}{3}$, $f_2(0) = 1.688$, $\inf_{0<x<a} f_2(x) = 1.393$, occurring at $x = 0.4179$, and $f_2(x_1) = f_2(0)$ for $x_1 = 3/(2 + a^{-3})$, $= 0.5581$ for $a = \frac{2}{3}$.

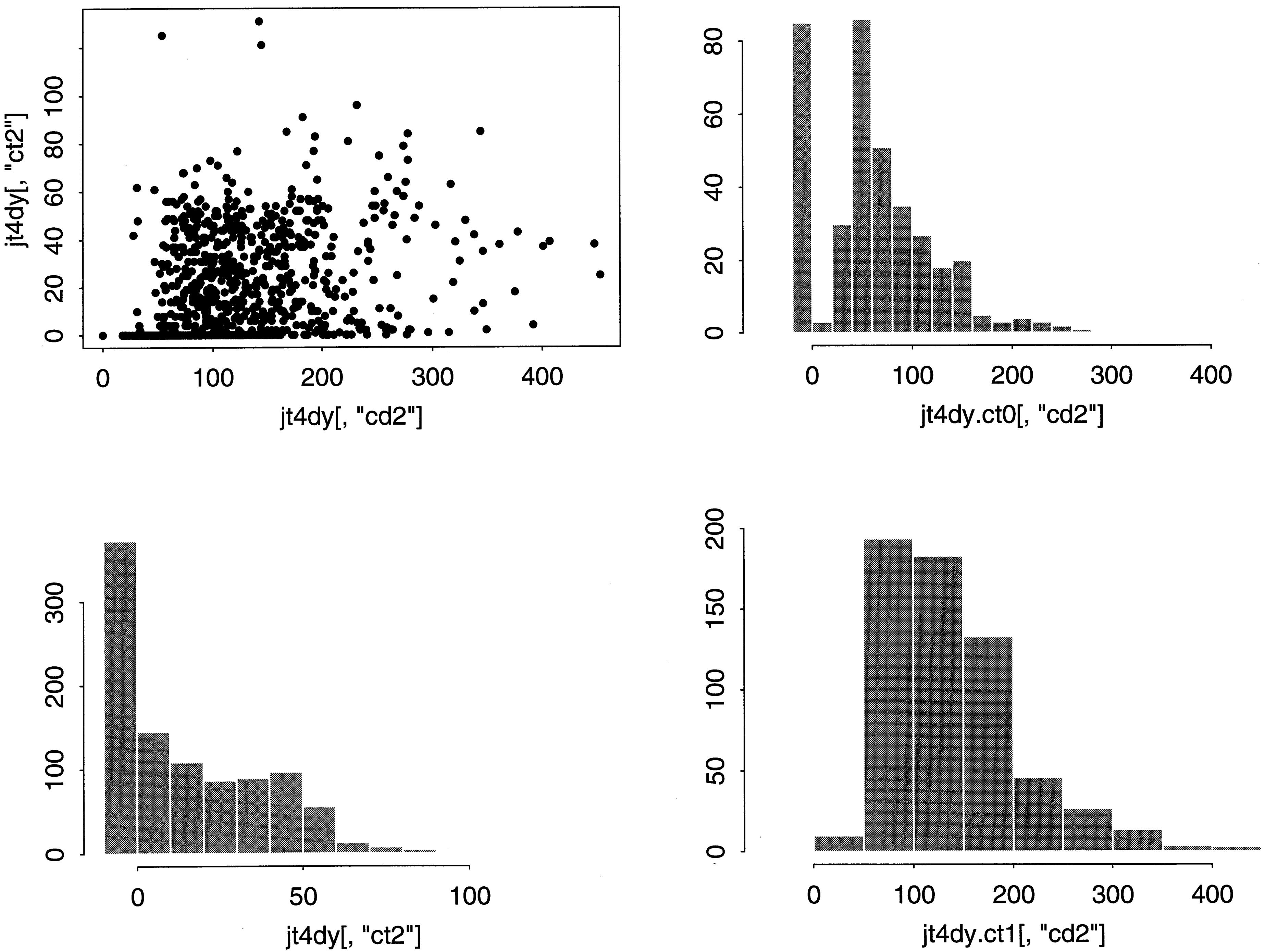

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Figure A.1

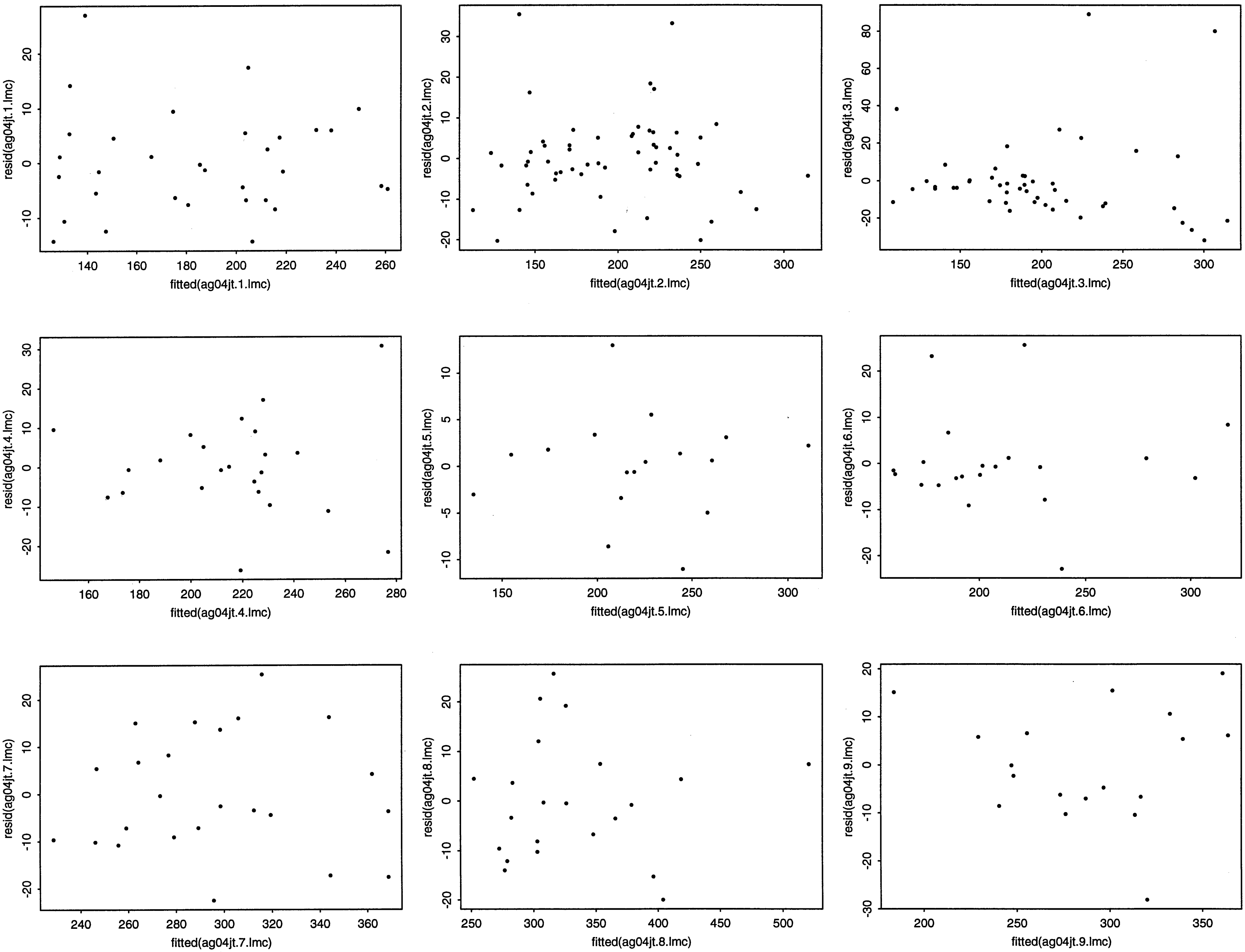
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Figure A.2

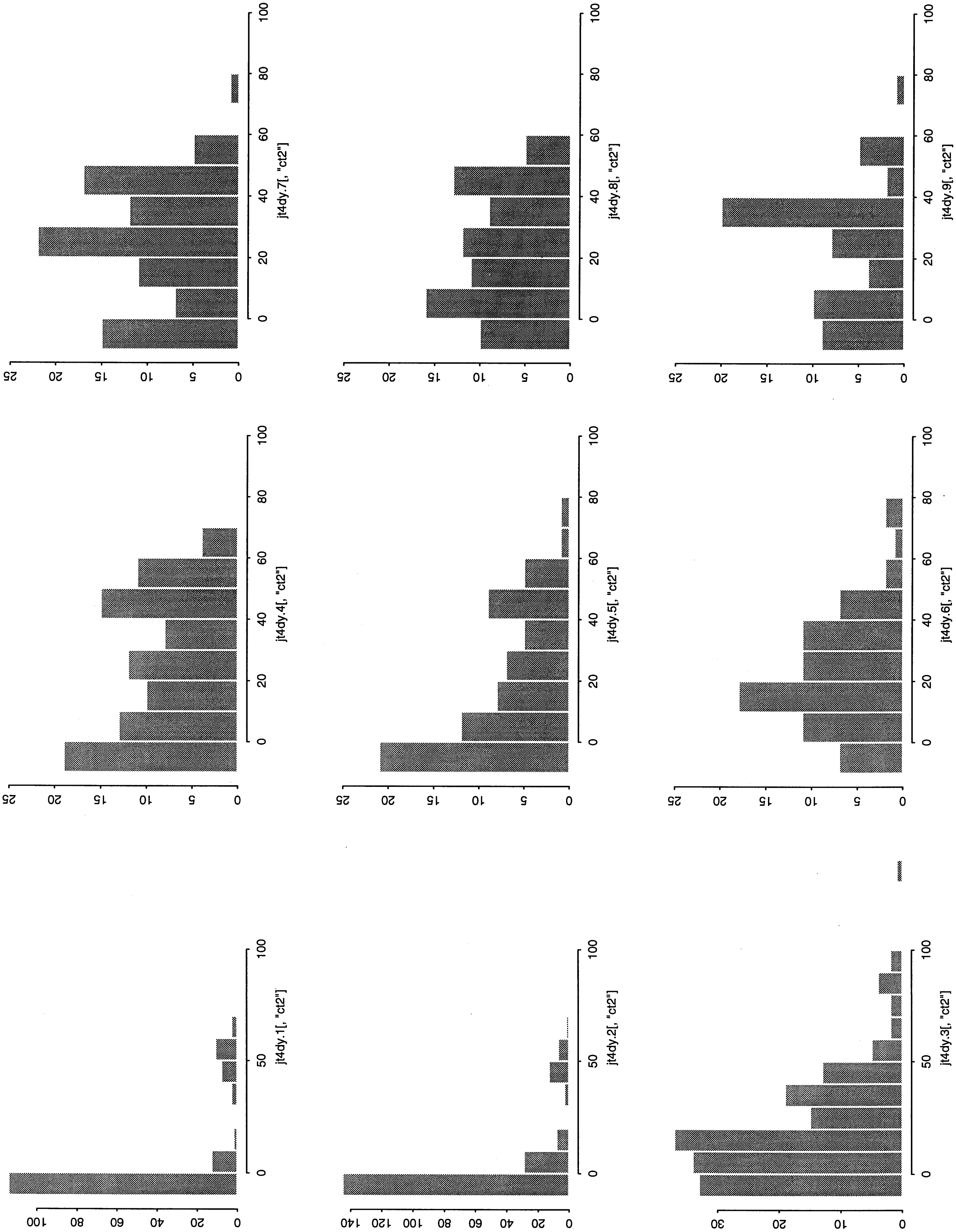

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Figure A.3

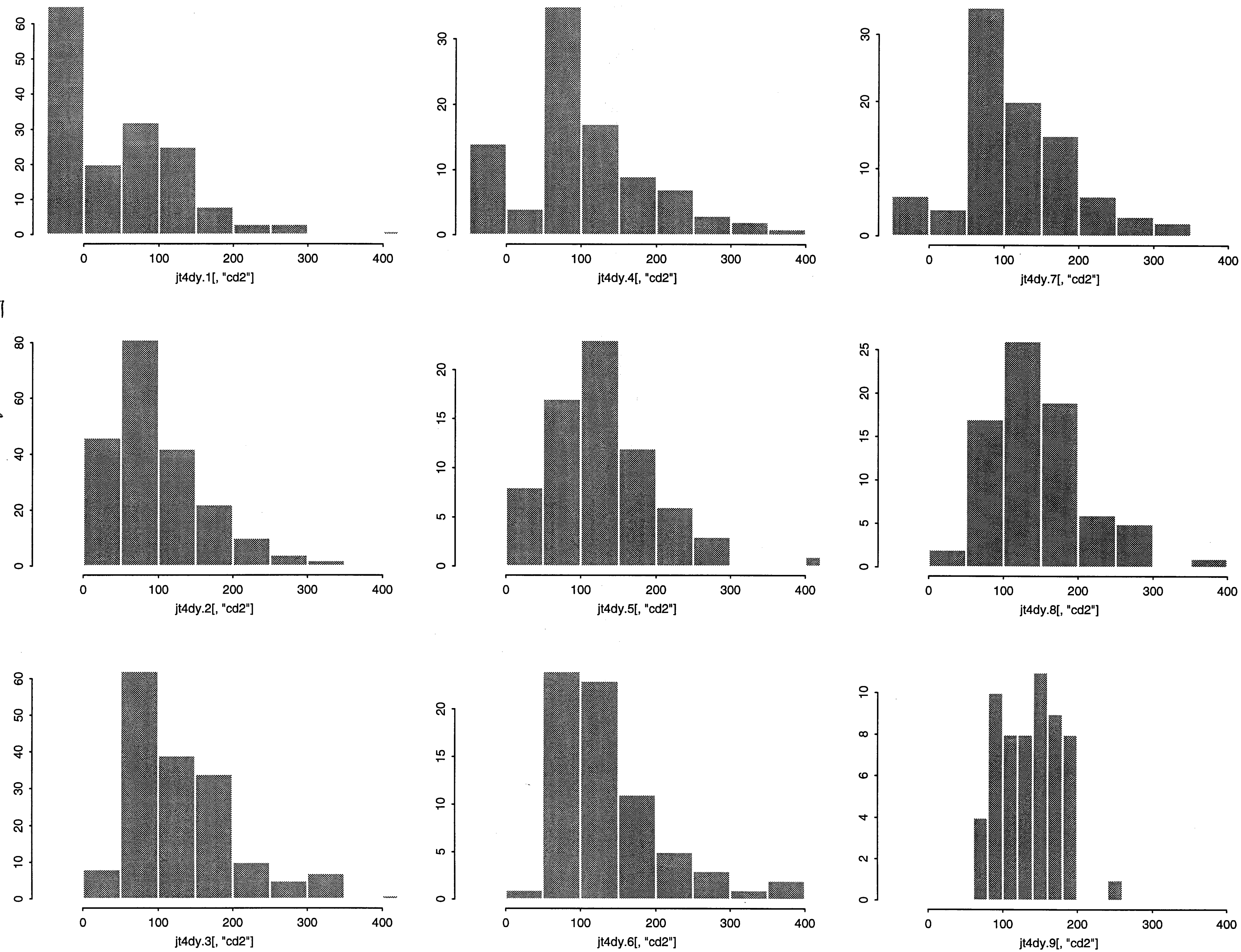
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Figure A-4

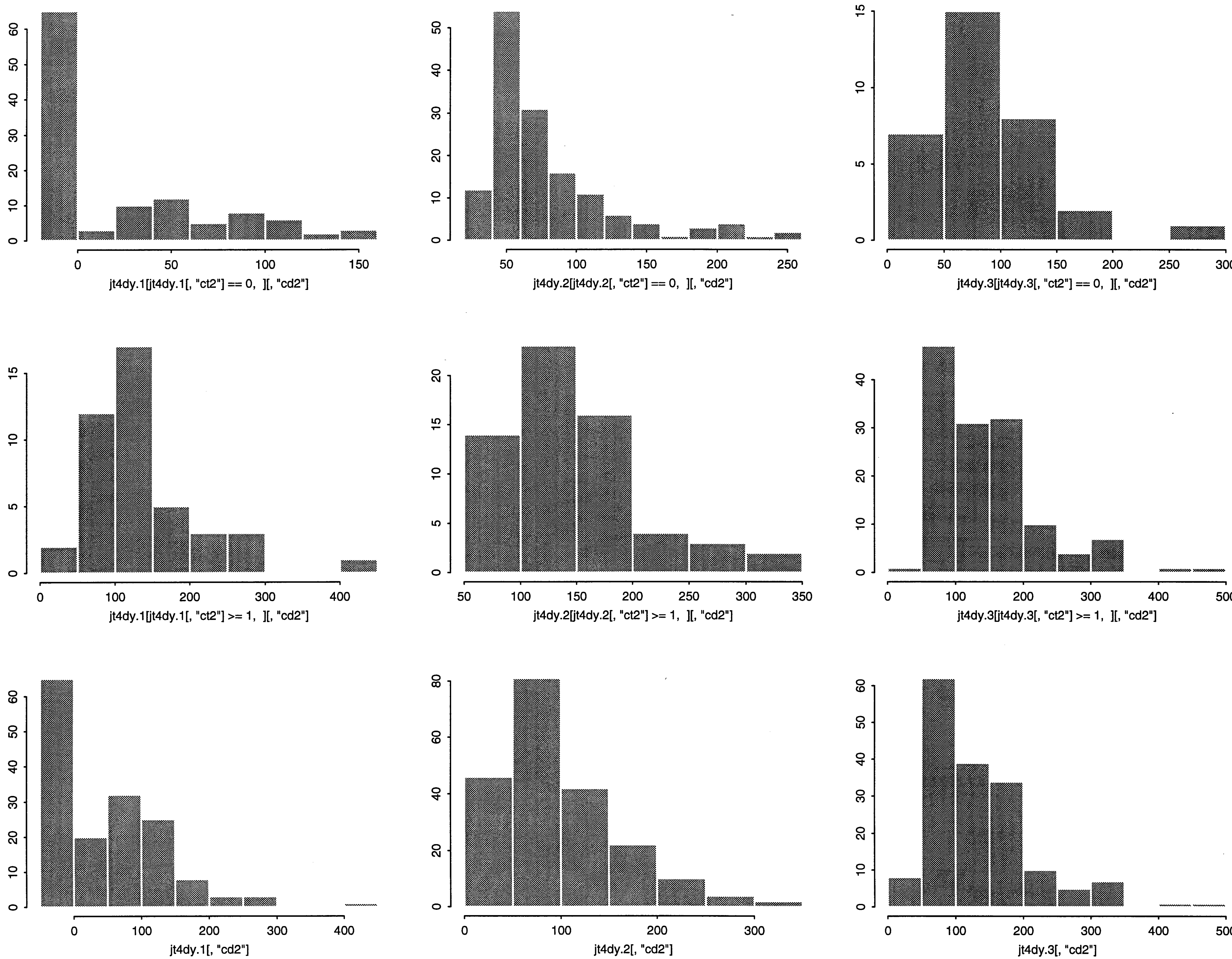
jt4dy.1[jt4dy.1[, "ct2"] == 0,][, "cd2"]
jt4dy.2[jt4dy.2[, "ct2"] == 0,][, "cd2"]
jt4dy.3[jt4dy.3[, "ct2"] == 0,][, "cd2"]
jt4dy.1[jt4dy.1[, "ct2"] >= 1,][, "cd2"]
jt4dy.2[jt4dy.2[, "ct2"] >= 1,][, "cd2"]
jt4dy.3[jt4dy.3[, "ct2"] >= 1,][, "cd2"]
jt4dy.1[, "cd2"]
jt4dy.2[, "cd2"]
jt4dy.3[, "cd2"]

Figure A.5