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Assignment-type Models

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# Arterial Link Travel Time Estimation: Probes, Detectors and Assignment-type Models

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## Abstract

Several methods of link travel time estimation on signalized urban arterials are compared. The difficulties associated with each method are identified and in some cases remedies are suggested. In particular, it is shown that the estimation of link travel times largely reduced to estimating stopped delay, and that, besides probes, most methods have difficulties estimating stopped delays.

## 1 Introduction

Estimation of link travel times is an integral part of Intelligent Transportation Systems (ITS) and specifically Advanced Traveler Information Systems (ATIS). Using the authors' experience with the ADVANCE system, in this paper we examine the usefulness of various methods that are used to obtain link travel-time estimates.

The most direct method of obtaining link travel times is through the use of probes, vehicles equipped to identify when they enter and leave links and to compute the time between these two events. Clearly, this process can be carried out manually. Moreover, vehicle travel times can be measured by direct observation from outside the vehicle as well. Using AVI technology, one can determine when a specific vehicle passed two points and thereby obtain its travel time. A similar process can also be carried out using video technology. For the purposes of this paper, these various methods are indistinguishable. When we measure the link travel times of individual vehicles directly, from within the vehicle or from outside, we shall refer to the vehicle as a probe and to the link travel times it reports as probe observations, probe reports or probe travel times.

A very common method for estimating travel times, particularly on highways, is detectors. In this paper, we will consider loop detectors, but the discussion below can relate to several other types of detectors. Loop detectors primarily provide information on volumes and occupancies over what we will refer to as reporting intervals. Volumes are the number of vehicles passing over a loop detector over a reporting interval; while occupancy is



the percentage of time a vehicle is above the detector. Reporting intervals typically range from one minute to five minutes. Volumes and occupancies are used to estimate speed on expressways [a job made much easier by using two detectors in close proximity] and have been used to estimate link travel times on arterial streets.

A third method is sometimes used to estimate travel times, based on assignment-type models of the four-step Urban Transportation Planning System (UTPS) process. These models are widely used to estimate link volumes and since they contain functions [called cost functions] to convert volumes to travel times, they may be used to estimate link travel times. However, one difficulty is that they give average travel times over long periods of time [e.g., a day] which we call assignment time periods. This difficulty has been addressed by constructing a succession of assignment-type models each for short time periods during each day. When this is done the models have been called dynamic or, more properly, quasi-dynamic. In this paper, when we refer to such models we call them short time-interval assignment-type models since dynamic traffic assignment covers a wider range of models. However, our comments below apply to a number of these models.

The following relation is commonly assumed for arterials:

$$\text{link travel time} = \text{cruise time} + \text{stopped delay} \quad (1)$$

The implication is that upon entering an arterial a vehicle travels at a relatively constant speed until it stops [if it does] because of a red light or a queue at the end of the link. The time spent while it travels more or less unimpeded is called the cruise time, while the stopped delay is the time spent stopped or traveling very slowly in a queue. As long as queue lengths are a very small proportion of it, cruise times depend mainly on driving styles and for most practical purposes can be taken to be a constant.

In Section 2, we empirically verify that the relation (1) indeed holds. From (1), it follows that variations in link travel times are largely due to variations in stopped delay. Stopped delays depend on volume and capacity to almost the same extent and also on progression. Therefore, volume alone is unlikely to be a good predictor of link travel times. Of course a great deal depends on the placement of the detector. These and other related issues are explored in Section 3. Overall, our conclusion is that a very large number of detectors would be necessary to get reasonably good estimates of link travel times. Moreover, fairly short reporting intervals would be necessary.

Section 4 is devoted to travel time estimates from short-time interval assignment-type models. Apart from the difficulties arising from the fact that these models estimate travel times from volumes and capacity, even the estimation of volumes present difficulties. A key one is the fact that the capacity of upstream intersections provide an upper limit for the volume for downstream links and intersections. In our judgment, this happens often enough that link travel time estimates would be seriously affected unless this is taken care of and it is not entirely clear to us how difficult it would be to treat this problem within the context of assignment-type models—unless some type of vehicle-by-vehicle simulation is incorporated.

Probes also present some difficulties. First unless the proportion of probes within the total vehicle fleet is large, data scarcity is a problem. Even with large numbers of observations [using video, say], the fact that probe-reported travel times are not statistically independent, keeps the variance very high. This issue is discussed further in Section 5 where two methods of dealing with this difficulty are also described. Our conclusions are given in Section 6.

## 2 Link Travel Times Models

As a part of the evaluation of the ADVANCE project, a number of probe vehicles were driven by paid drivers over a study route (Figure 1). The route consists of twelve links, many of which are on one major arterial Dundee Road on Wheeling, Illinois (suburban Chicago). These probes measured link travel times as well as congested times—which was the time spent on a link moving at a speed less than 2 meters per second or approximately 4.5 miles per hour. Other data were also collected. Details of the data collection effort, an assessment of the quality of probe reports and other pertinent information have been published elsewhere [1].

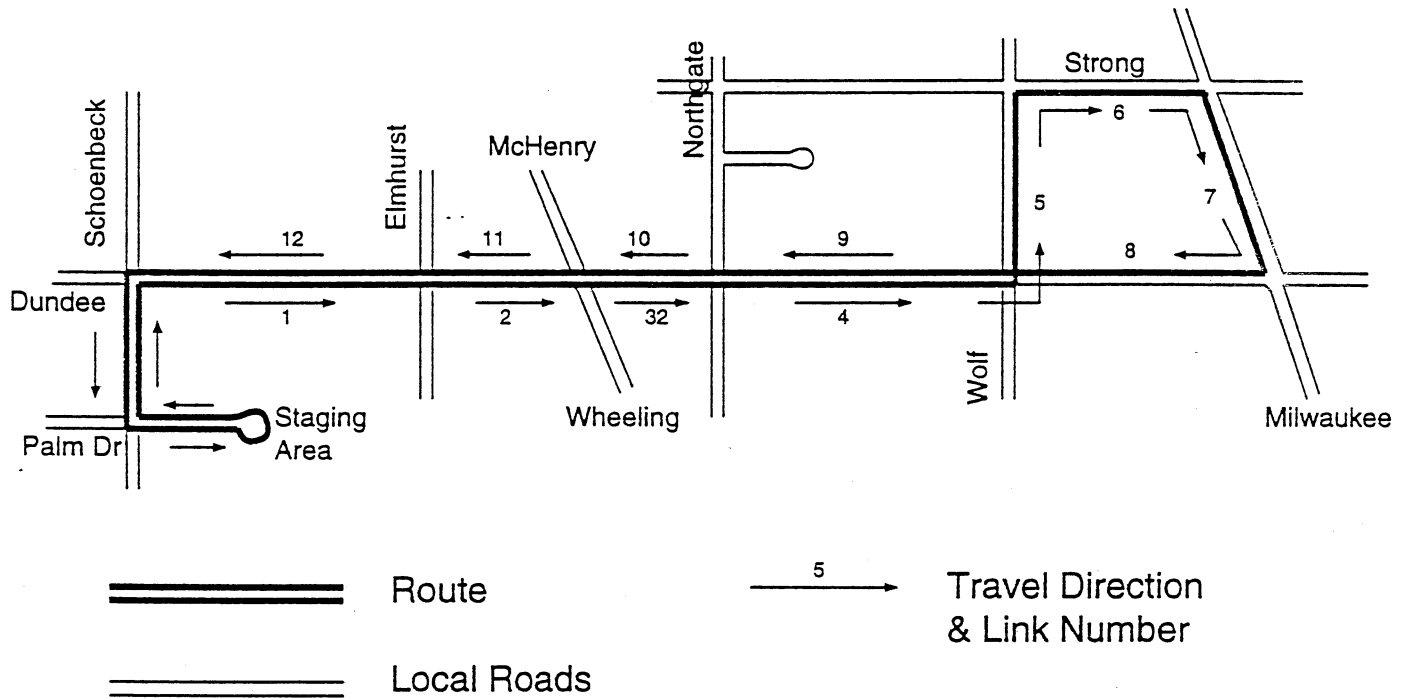


Figure 1: Probe Data Collection: Long Route

The probe vehicles did not measure precisely stop delay but rather the number seconds the vehicles travel at speeds less than two meters per second including being stationary. The relationship between travel time and this variable is considered in two least squares fitted models:

$$TT_i = \beta_0 + \beta_1 CT_i + \epsilon_i \quad (2)$$

$$TT_i = \beta_0 + \beta_1 CT_i + \beta_2 CTD_i + \epsilon_i \quad (3)$$

where  $TT_i$  = the travel time on link  $i$  (in seconds)  
 $CT_i$  = the congested time on link  $i$  (in seconds)  
 $CTD_i$  = 1 if there is congested time, otherwise it is zero  
 $\epsilon_i$  = the error term

Figure 2 show plots of travel time and congested time collected over a ten-week period on two links. Each figure has two lines. The upper line through the data is the regression line and the lower line shows where the congestion distance equals the travel time. The vertical distance between the two lines is the travel time on the uncongested portion of the link.

The first figure shows 2231 trips over Link 7, the most congested link in the study route. It is only 520 meters long but as can be seen on the figure there are numerous passes over this link that require more than five minutes. The figure also shows that for most drivers the congested time is the major determinant of link travel time. The only exceptions are several dozen passes clustered in the bottom left hand corner of the graph illustrating rather long travel times given the relatively short congested times. This can be attributed to driver behavior.

The second figure shows the relationship between congested time and travel time for the other extremely congested link on the study route, Link 9. This link is longer, 856 meters, but it has far fewer reports over five minutes. Also the slow drivers can be seen, now over a longer period, up to 150 seconds.

It can be seen that the regression line for Equation (2) seems to have a slope greater than one, especially for Link 9 and therefore there are a number of observations below the regression line near the the upper end of the line. This phenomenon can be seen in Table 1 which shows parameter estimates and  $R^2$ 's for selected study links. The congested time coefficients are greater than one, a rather counterintuitive finding for uncongested links. One would expect that on uncongested links the with congested distances the remaining travel time would be short. More on this below.

Also note that the  $R^2$ 's are quite high. In fact they fall below 0.9 only on uncongested links that do not have much variation in average link travel times over the day. This is therefore due more to the definition of  $R^2$  than a lack of fit ([2], Chapter 1, 1990).

Table 1: Parameter Estimates and  $R^2$  [from Model (2)]

Link	Length(m)	$\beta_0$	Std. Error	$\beta_1$	Std. Error	$R^2$	Obs.
1	817	55.2	0.218	1.04	0.009	0.78	3957
2	457	32.9	0.194	1.12	0.006	0.89	3980
3	403	27.1	0.151	1.22	0.010	0.86	2231
7	520	43.1	0.345	1.08	0.003	0.98	2231
8	660	48.3	0.213	1.11	0.006	0.95	2039
9	856	57.1	0.236	1.23	0.006	0.96	2017

If we interpret congested time as stopped delay and the intercept as cruise time, then we see that the model holds pretty well under 'real life' conditions. A better model is

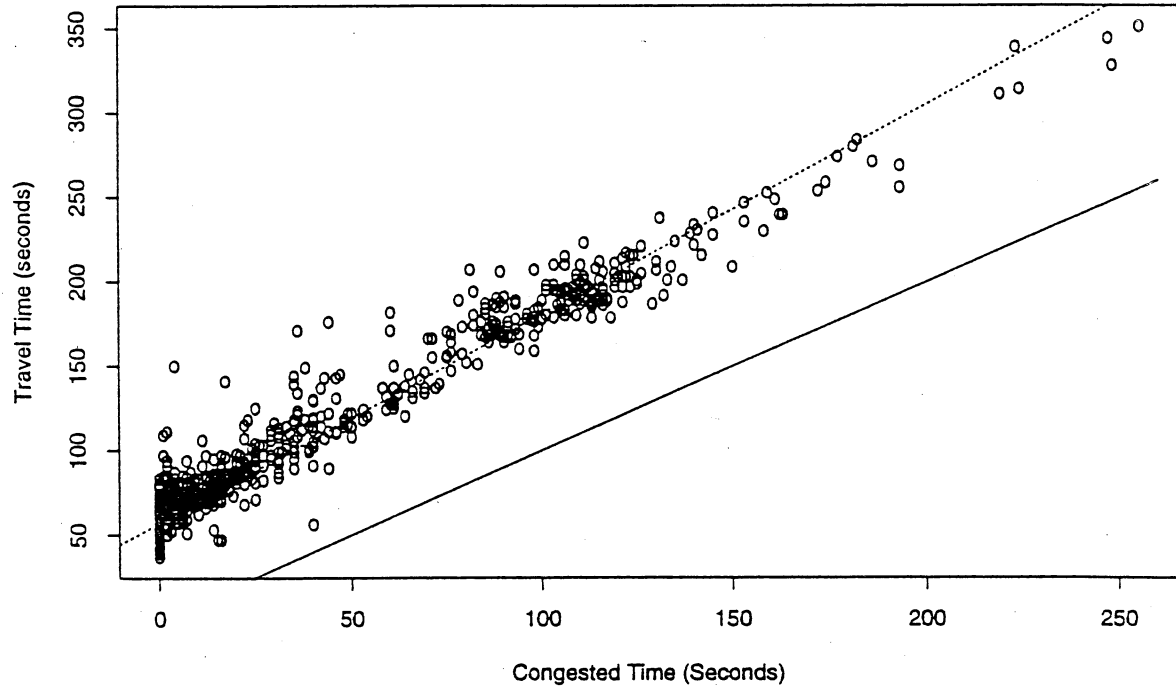
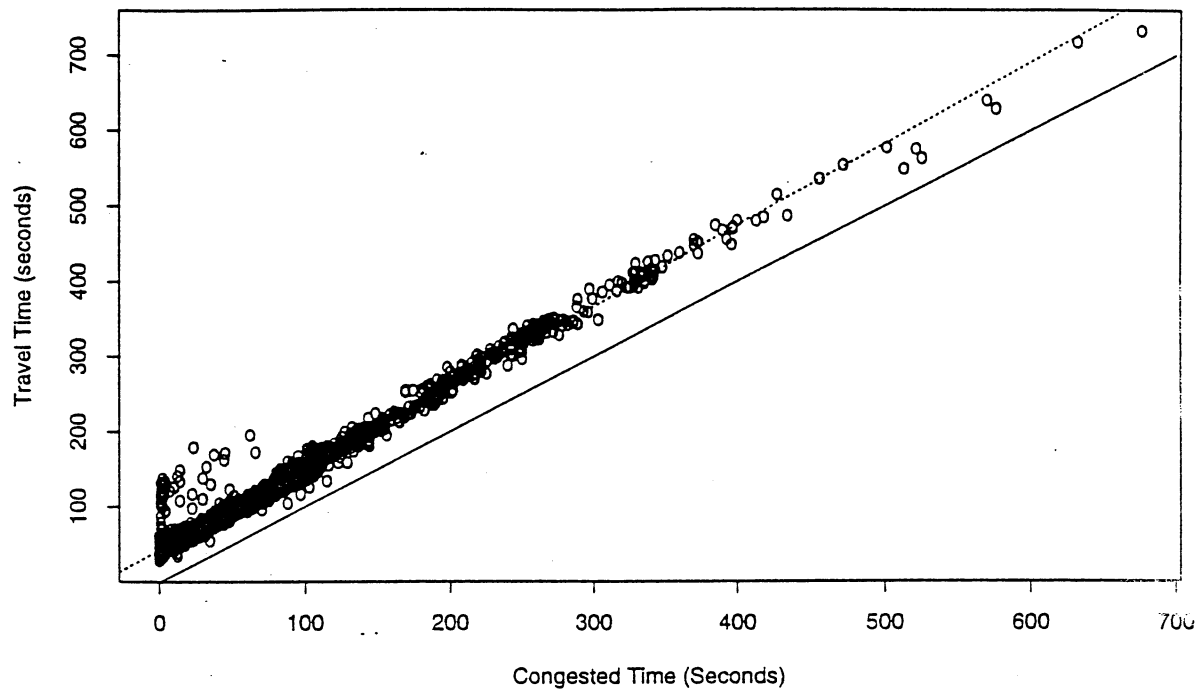


Figure 2: Travel Time vs. Congested Time: Link 7 (Top), Link 9 (Bottom)

achieved, however, by adding a dummy variable to signify whether congested time was recorded (Table 2). It might be mentioned that a more detailed model [i.e., with more variables] has been fitted by Daly [3] (see also [4]) to these same data.

Table 2 shows a substantial improvement in the  $R^2$ 's values for all links but particularly the first three. These have low levels of congestion, low travel times, and therefore a more significant stop delay effect.

The important point in this section is that there is a strong relationship between congested time and travel time.

Table 2: Parameter Estimates and  $R^2$  [from Model (3)]

Link	$\beta_0$	Std. Error	$\beta_1$	Std. Error	$\beta_2$	Std. Error	$R^2$
1	50.6	0.338	0.80	0.014	14.2	0.575	0.83
2	29.3	0.254	0.89	0.010	16.9	0.557	0.93
3	25.4	0.199	0.98	0.016	11.4	0.517	0.91
7	42.1	1.161	1.09	0.004	0.4	1.260	0.99
8	44.6	0.466	1.12	0.008	6.0	0.630	0.96
9	55.0	0.455	1.16	0.006	17.1	0.902	0.98

### 3 Detector Data

Detectors have been installed on arterials to help control traffic signals and monitor traffic and they are usually placed fairly close to intersections. If the queue formed at the intersection oscillates above the detector, then the detector readings can be used to estimate average link travel times with moderate accuracy. However, if a cycle failure occurs and the queue is so long that it always extends over the detector, the detector will give similar readings regardless of the length of the queue.

This fact is illustrated by Figure 3. The first figure shows occupancy (as a percent) for Link 7 on July 6, 1995 and it tops off at 60%. When occupancy exceeds this level this reading no longer is a useful measure of the magnitude of congestion beyond this level. The second figure represents the volume for the same day. While the volume does not stop abruptly as in the case of occupancy, when the link becomes exceedingly congested then the detector begins to have difficulty in distinguishing between a vehicle and two vehicles very close to each other. In these cases the detector underestimates the volume [5]. These highly congested cases and cycle failures are clearly more important for ATIS than link travel times under lower congestion levels.

Now, suppose we extended the distance of the detectors from the end of the link to a point where the queue never extends over the detector. If cruise speeds are more or less constant, volumes and occupancies essentially yield the same information. Clearly, volume is an important component for the determination of travel times, but good estimation requires information both on capacity and on progression.

Capacity requires knowledge of green phase times and if signals are demand actuated, as they would be typically when detectors are present, green times would vary and this information needs to be communicated to the computer being used for travel-time estimation.



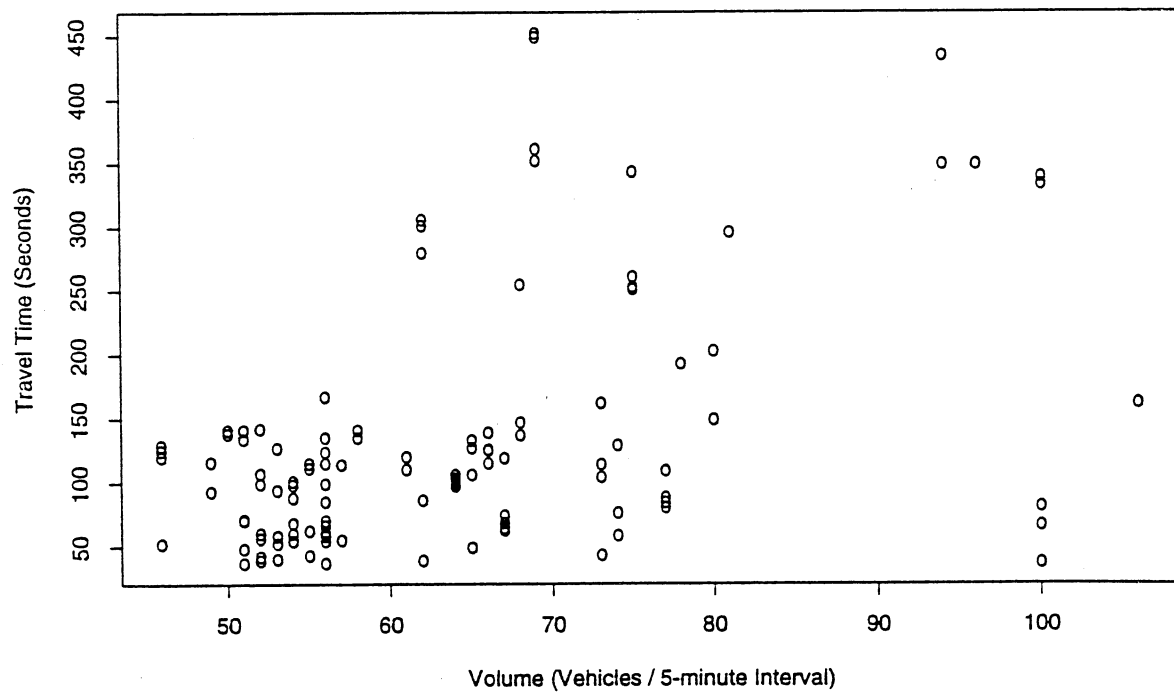
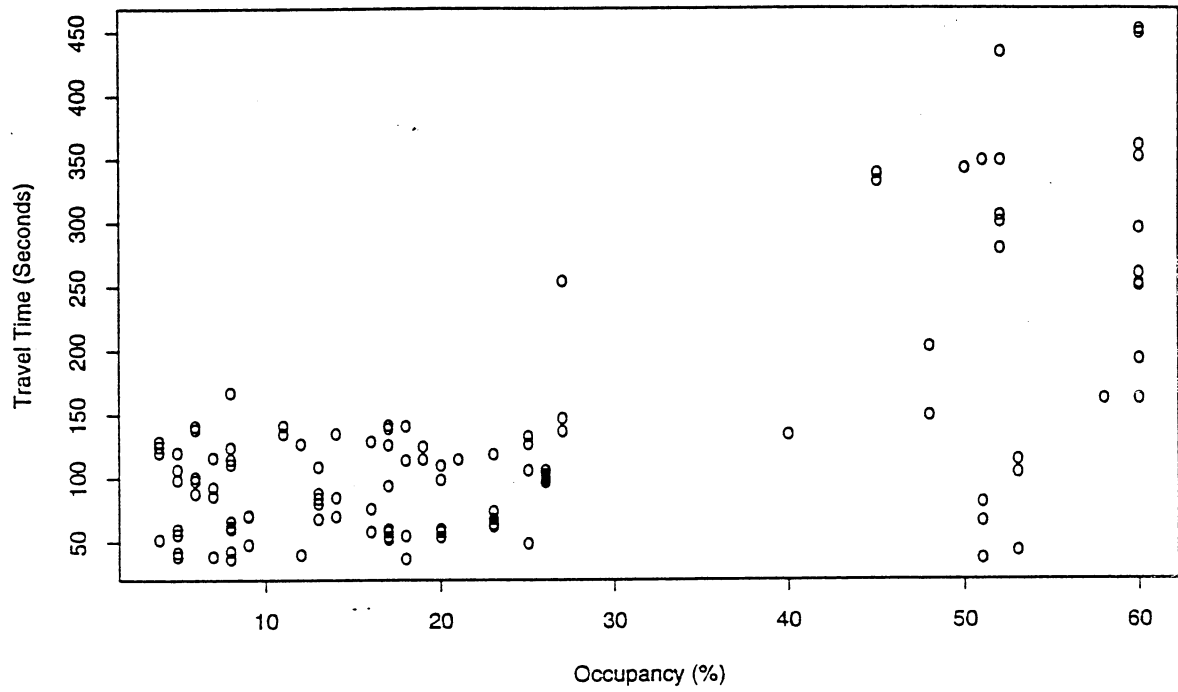


Figure 3: Relationship between Travel Time and Occupancy (Top) and between Travel Time and Volume (Bottom)

The actual times vehicles arrive at the intersection is also critical. For example, under moderate congestion, vehicles arriving during the later part of a green phase would encounter almost no stopped delay, while a vehicle arriving at the start of a red would encounter a much longer delay.

Figure 4 illustrates the relationship between the start of the green phase and link travel time on Link 11, August 4, 1995 [6]. This link includes a signalized intersection at only each end. The signal cycle begins on a red while generally the green phase begins at approximately the 72-second mark, on some occasions the leading, opposing left-turn arrow is not activated and the green begins at the 60-second mark.

Therefore getting good estimates of arrival times would require very short reporting times, which in turn could involve fairly wide band communications — although recent advances in encryption technology can substantially alleviate the difficulties.

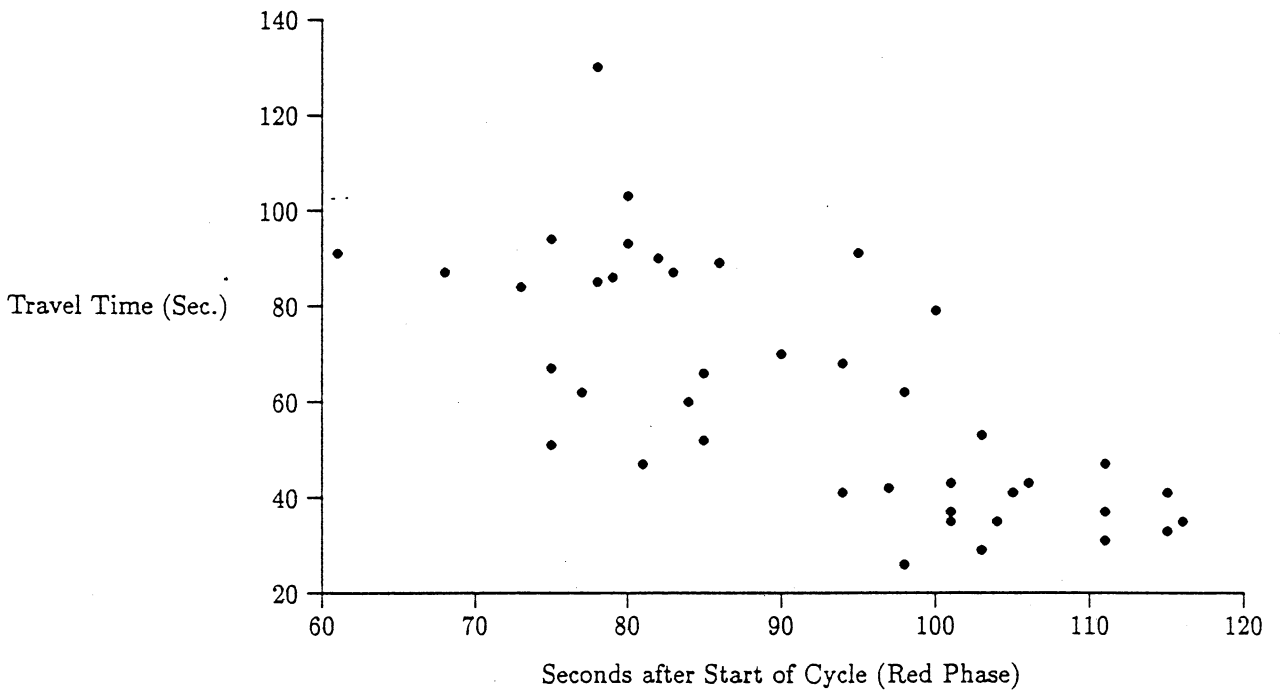


Figure 4: Relationship between Travel Time and Start of the Green Phase

Given that most traffic on a typical link enters it as through movements from upstream links and most of this traffic typically enters right after the upstream light turns green, one might be able to get an understanding of arrival times on the study link from progression. However, this too would require knowledge of traffic signal timings. Another difficulty with this approach is that if volumes fluctuate, the assumption that this increase is solely due to increases from vehicles entering by means of through movements might not be valid.

Thus there are considerable difficulties in estimating travel times using detectors. Perhaps the only way in which this can occur successfully is by detectorizing all lanes on all links, using very short reporting intervals and having information on traffic signal timings. In that case volume readings from upstream links can give us volume information for downstream links, which together with signals timings could give us all that we would require

for effective estimation of link travel times.

## 4 Assignment-Type Models

Assignment-type models depend on the assumption that every driver, with a given origin and destination and with full knowledge of the system chooses the route that minimizes his/her travel time. These models require as input volumes from each origin to each destination [O-D volumes] and they are very useful in long range transportation planning for which they were originally created. However, producing very detailed estimates of O-D volumes for a specific day presents difficulties. These difficulties are being addressed.

These models estimate link travel times from volumes — via a relationship called a cost function. Such functions tend to be closed form algebraic functions, although perhaps better ones can be developed using the discussion from the next section.

However, there is much more serious drawback with these models. Assignment-type models assign the volume on every route to all the links comprising the route. These are average volumes over the appropriate time interval. Sometimes, and not uncommonly, under congested conditions, upstream intersections act as filters for downstream links so that the latter never achieve the volume to capacity ratios that the upstream link achieves, even though the two links might have the same capacities and might even be identical in other ways.

This fact is illustrated in Figure 5 using typical cost functions. These diagrams assume that the total volume on the upstream link over the three time intervals, within the assignment time period, are equal [volumes  $A + B + C = A' + B' + C'$ ]. In the upstream link we have three separate consecutive volumes [A, B and C] leading to three different travel times [a, b and c] with an average assignment interval travel time of  $\alpha$ . In period A there is a moderate traffic volume followed by a major surge of traffic in B and then again a much lower volume in period C.

In the downstream link, for simplicity we assume that traffic only enters from the upstream link, the interval volume would be the same but the volumes during each green phase are dependent upon the capacity of the upstream intersection,  $B'$ . The downstream average travel time  $\alpha'$  is quite different from  $\alpha$ ; all individual volumes are less than  $\alpha$ . If the congestion on the upstream link is so great that for the entire assignment time period the volume on the upstream link is larger, the average travel time on the upstream link would be even larger.

Therefore, the average travel times on the two links would be different although they have the same capacity. Moreover, a good estimate of this average travel time would require a fairly precise understanding of the volume variations on it or one the upstream link. If the assignment interval is small, these differences could be even larger. Cost functions do not reflect this fact and having them do so could be very difficult unless one supplements assignment type models with vehicle-by-vehicle simulation models e.g., of the NETSIM variety.

Some empirical data comparing assignment-type models with corresponding observations from twenty-five days of data collection by probes is in Table 3[7]. This table shows model estimates for both the off-peak and the peak period. The model performs best for Links 5 and 6, two off the major arterial portion of the study route. On the rest of the study route the model perform less well. Construction on a major parallel arterial may have

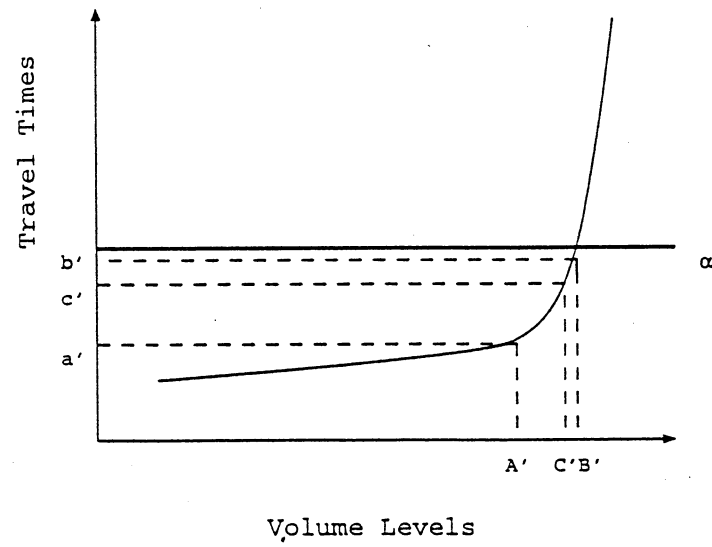
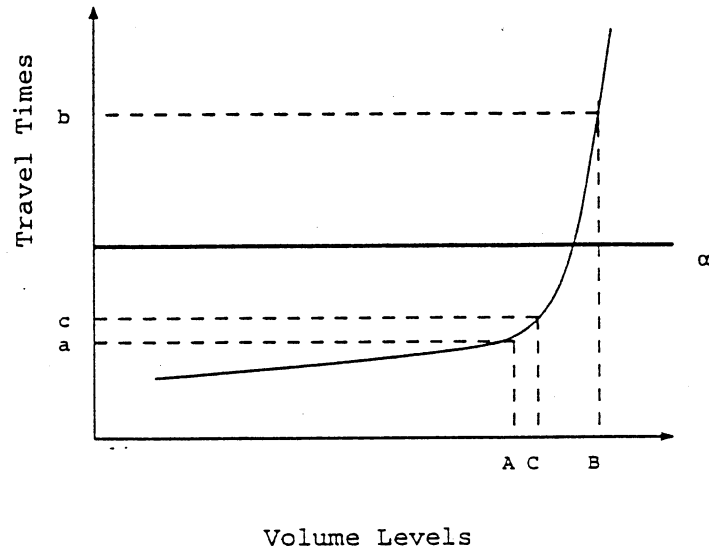


Figure 5: Relationship between Travel Timein and Volume in the Upstream Link (Top) and the Downstream Link (Bottom)

diverted traffic onto these links and there was some population growth in this area which was not provided as input. Still, the differences are sufficiently great that raise questions about the utility of this approach.

Table 3: Model and Probe Travel Time Estimates (in Seconds): Off-Peak and Peak

	Cruise	Model Data	Probe 1pm-4pm		Model Data	Probe 4pm-6pm	
Link	Time	1pm-4pm	mean	stdev	4pm-6pm	mean	stdev
1	45.9	56	75.6	22.6	54	70.8	23.4
2	29.3	36	43.8	30.1	34	58.8	33.8
3	22.6	76	30.1	16.1	79	41.5	19.5
4	54.9	64	95.4	33.1	65	119.1	47.1
5	32.7	33	36.5	4.1	33	38.0	7.5
6	43.3	44	45.3	9.7	53	56.8	26.6
7	33.4	47	107.4	52.0	56	225.4	120.8
8	42.3	55	55.6	17.6	44	104.6	73.5
9	48.1	105	64.0	30.5	175	198.8	107.6
10	25.8	32	60.3	32.7	34	82.2	25.4
11	25.7	35	56.6	28.9	57	52.7	31.1
12	45.9	61	74.9	23.6	46	85.9	35.7

## 5 Probes

Probes can overcome some of the difficulties with detectors but the limitations of probes also needs to be discussed. Link travel times for The variation in link travel times even within the same cycle (see Figure 4) is a problem. Under conditions of no cycle failure, a vehicle arriving at an intersection at the end of the green time experiences little delay and hence shorter link travel times than one that arrives at the start of the red. This is obviously true for probes as well. But the problem gets substantially compounded by the fact that travel times of vehicles are not statistically independent.

One effect of this is that many of the statistical formulæ one takes for granted are no longer valid. A more important effect is that variances of estimates based on probe reports increase. The issue has been dealt with in detail in Sen and Thakuriah [8] [see also [9]. Figure 8 and 9, showing plots of standard errors of travel-time means against number of probes per five-minute interval for two different links, are taken from that paper. The plots clearly show that standard errors do not go to zero with increasing probe deployment rates as would be the case had the reports been independent. Indeed after certain levels of probe deployment, increasing deployment levels seems to have little effect on standard errors. We remind the reader that while we make this case for probes, the fact holds for estimates based on any type of measurement of travel times of individual vehicles.

While this is a serious problem with data on individual vehicles, it is not entirely unsurmountable. We present two methods below. The first is easy to implement and essentially allows us to live with the problem. The second approach actually largely corrects the problem but requires information on signal timing.

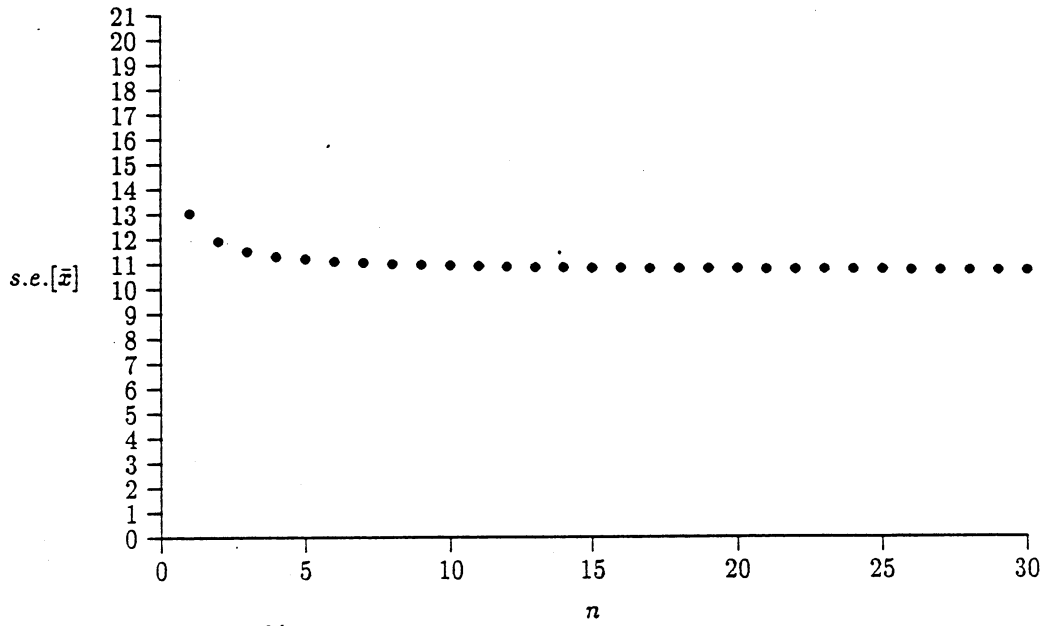


Figure 6: Relationship between Standard Error  $s.e.[\bar{x}]$  and Frequency  $n$  of Probes on Link 3 during the Peak Period ( $x$  is the link travel time)

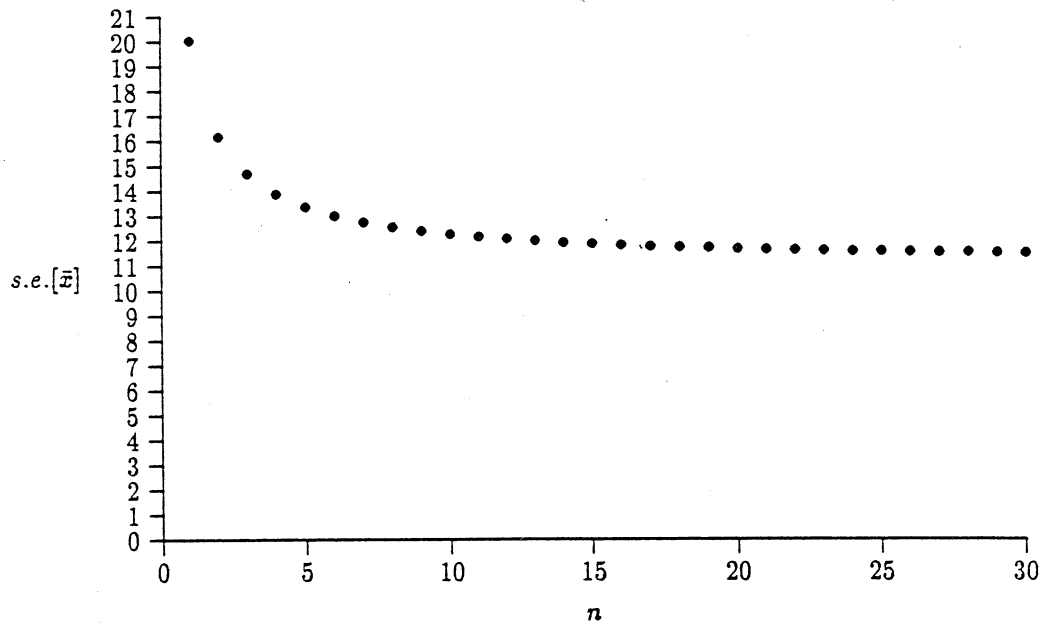


Figure 7: Relationship between Standard Error  $s.e.[\bar{x}]$  and Frequency  $n$  of Probes on Link 11 during the Peak Period ( $x$  is the link travel time)

## 5.1 Static Estimates

The first method is akin to that used in ADVANCE. For every interval for which we seek an estimate of travel time [call it a TT-interval], static estimates are constructed [10]. The interval may be as short as ten minutes when traffic conditions change and such as at the beginning of the peak period to several hours when traffic is light. The static estimates would be averages of travel times for that TT-interval, over several days. These would be estimates of ‘normal driving time’ for the link for the appropriate TT-interval. Correlations between probe reports from one day to the next do not appear to be high and these estimates seem to have very small variances if enough days are averaged over. Then the travel-time estimate for a specific day would be computed as follows:

- Use the static estimate if the estimate for the specific day does not differ from it significantly, and
- Use the specific day estimate otherwise (communicated from a central computer).

Thakuriah and Sen [11] ( see also [12]) have shown through a large-scale simulation that estimates obtained in this hybrid manner perform extremely well when used for route guidance.

## 5.2 Signal Timing and Probe Departure Time

As mentioned above, the second method actually reduces the problem. In order to describe it, we first examine the causes of the covariances. Figure 8 illustrates the covariance between link travel times as plotted against headways; i.e., suppose each point in the plot is designated as  $(x, y)$ , then  $y$  is the estimated covariances between a pair of probe observations and  $x$  is the corresponding difference in times of departure for two consecutive vehicles from the link. Not only are covariance easily seen, but it is also obvious that these covariances, as a function of headway have a cyclical pattern. This pattern is consistent with the signal cycle.

If we identify, as we have before, congested times with stopped delay, then

$$\text{link travel time} - \text{congested time} = \eta$$

is an estimate of cruise time (on the uncongested portion). Figure 9 shows a plot of estimated covariance between pairs of  $\eta$ 's against corresponding headways. While there does seem to be some pattern here, it is not quite as obvious as before. note that the vertical scales of Figures 8 and 9 are very different. Indeed the strongest pattern seems to be cyclical, showing that cruise times are also affected by traffic light cycles — but only slightly. While a vehicle following another is likely to have similar link travel times and therefore cruise times would appear intuitively to be correlated, given that probes tend to be widely separated and therefore the covariances between contiguous vehicles decay in the interval between two probes. At any rate, the association with the traffic light cycles seem to be dominant.

Seen this way, it becomes fairly obvious, why there is a strong effect on travel time estimates from probe reports. Notice that vehicles departing early in the green phase would usually have had longer delays than those departing late. If the TT-intervals are not coordinated in some way with traffic light cycles, some TT-intervals would include more ‘early greens’ than others [see Figure 12]. The average delay and hence average link travel

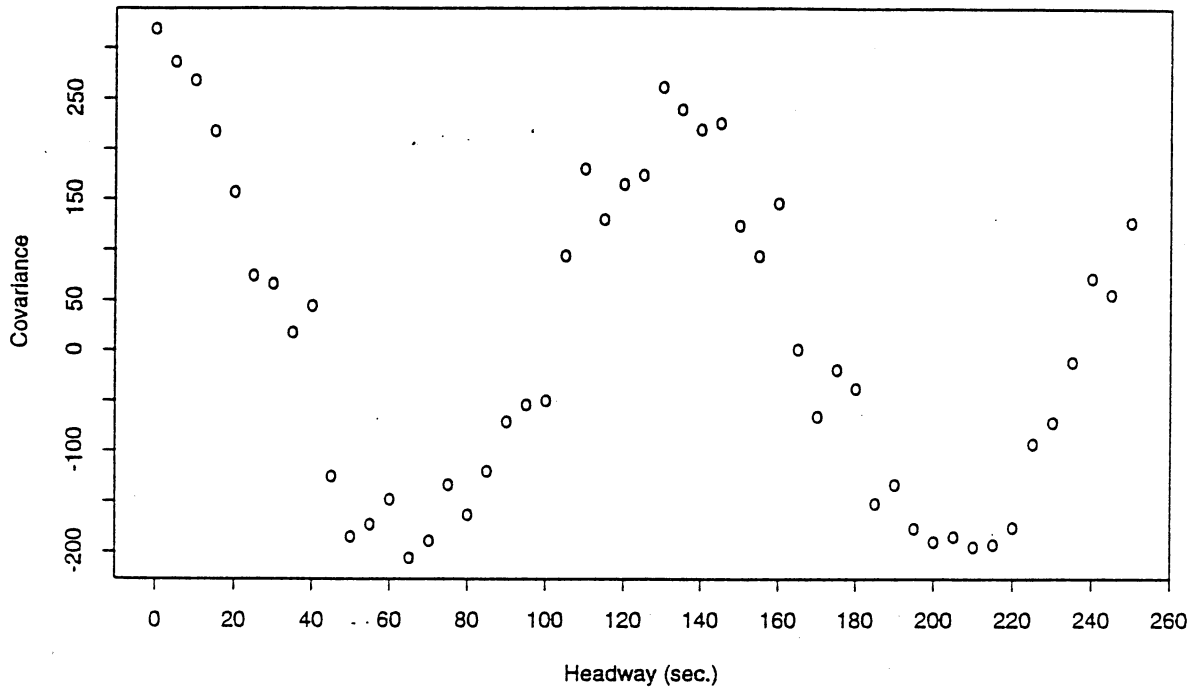


Figure 8: Covariance between Probe-Reported Travel Time vs. Vehicle Headway

times for TT-intervals with more early green would typically be higher than those with less early green. This fact would not be substantially affected by levels of probe deployment and offers a partial explanation for Figure 5.

It also offers a procedure for alleviating the problem: make TT-interval lengths be an integer multiple of cycle lengths when cycle lengths are constant. If they are not constant then TT intervals would need to include an exact integer number of cycles. This would require:

- precise knowledge of cycle timings and
- precise knowledge of the time each probe departs a link,

in addition to probe travel times.

However, apart from the problems of correlations between probe observations, a difficulty that can be alleviated, as we have just seen, probe data present an additional well known problem which we now turn to. A very important factor is coverage, i.e., the number of links being covered during an appropriate time interval. This issue was addressed in one of the first documents written for ADVANCE design [13], [14].

It should, however, be borne in mind that links not covered are mostly less traveled links. Such links are less likely to suffer from congestion and given the nature of the route choice algorithm used, are also less likely to form part of routes that are recommended to drivers.



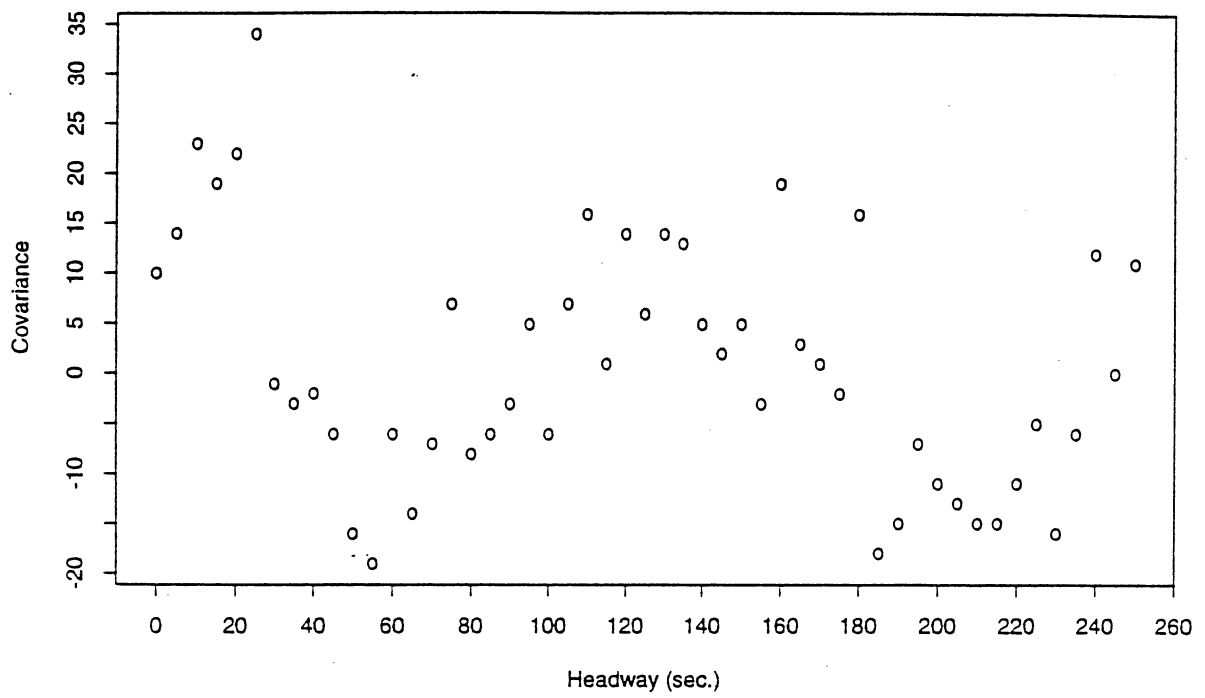


Figure 9: Covariance between Probe-Reported Travel Time - Congested Time vs. Vehicle Headway

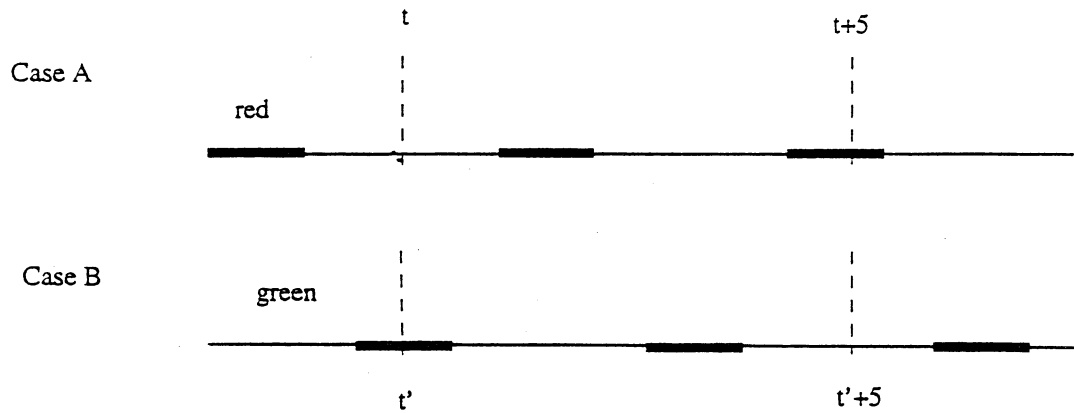


Figure 10: Relationship between Traffic Signal and Travel Times

## 6 Conclusions

Each of the means for estimating link travel time presented in this paper have their advantages and disadvantages. Future research can alleviate some of the disadvantages. If we had to make a recommendation we would have to make one under the warning given by Karl Pearson (1898, see also Rao[15], 1973, p. 284):

No Scientific investigation is final; it merely represents the most probable conclusion which can be drawn from the data at the disposal of the writer. A wider range of facts, or more refined analysis, experiment, and observation will lead to new formulas and new theories. This is the essence of scientific progress.

With this caveat, our current thoughts are that assignment-type models will require substantially more work before they can be used effectively in travel time estimation and for detectors to be effective, the level of deployment of detectors (large number of detectors reporting for short time intervals) and the communications required might be cost prohibitive. Probes, which represent the best way to record stop delay, also have difficulties associated with them. But these might be more easily alleviated and consequently, for link travel time estimation, we see this as the most promising method.

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