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Dynamic User-Optimal Route Choice Modeling of a Large-Scale Traffic Network[1]

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Abstract

Application and extensions of a dynamic network equilibrium model, DYMOD, to the ADVANCE Network are described in this paper. ADVANCE is a dynamic route guidance field test designed for 300 square miles (770 square kilometers) in the northwestern suburbs of Chicago. The dynamic route choice model employed in this paper is solved efficiently by Janson's dynamic algorithm. Except for a small portion of links, realistic traffic engineering-based link delay functions, instead of the simplistic BPR (Bureau of Public Roads) function, are applied to estimate link travel times and intersection delays for various types of links and intersections. Further, an expanded network representation is utilized. To this end, nearly 23,000 links and 10,000 nodes are modeled in this research. The time-dependent link flow, travel time, speed and queue spillback information are generated for the ADVANCE Network. The ADVANCE Network is divided into 447 zones, originally specified by the CATS (Chicago Area Transportation Study), to assign time-varying travel demand on the basis of CATS estimates for 1990. This is the largest dynamic route choice solution which has been obtained thus far, to the knowledge of the authors. The model has been solved on a CONVEX–C3880. Convergence and computational results are presented and analyzed.

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1 Introduction

Intelligent Transportation Systems (ITS), also known as Intelligent Vehicle Highway Systems (IVHS), make use of advanced technologies (such as navigation, automobile, computer science, telecommunication, electronic engineering, automatic information collection and processing) in an effort to improve the movement of people and goods on the road. ITS technologies have the potential to provide better travel information, easier and safer travel, improved network capacity utilization, less traffic congestion, improved traffic flow, energy consumption savings, quicker emergency response, faster freight deliveries and improved fleet management.

Within the framework of ITS, Advanced Traveler Information Systems (ATIS) can provide historical, real-time and predictive information to support travel decisions; this will in turn influence the travel choices of individuals and consequently improve the time and quality of travel (Ran and Boyce, 1994). Besides ATIS, Advanced Traffic Management Systems (ATMS) is expected to integrate the management of various roadway control functions including ramp metering, signal timing , and variable speed advisories to predict traffic congestion and provide alternative routing instructions to drivers. General roadway information can be broadcast to all drivers through AM/FM radio. Conversely, detailed information generated by ATMS can be transmitted to specific groups of drivers via in-vehicle route guidance systems. In consideration of the goals of ATMS and ATIS, a dynamic route choice model based on utility maximization route choice assumptions, is needed to perform traffic condition assessment and prediction and generation of route guidance. That is, a dynamic route choice model is an essential element for ATMS and ATIS evaluation and implementation.

The problem that is analyzed in this paper is stated as follows: Given time-dependent information on travel demand (in the form of origin-destination (O-D) matrices for short time intervals), distribute demands onto routes through the network based on (1) specified route choice behavior (e.g., user-optimal and/or system-optimal), and (2) time-dependent transportation supply variations. For a specific solution procedure, the interaction between transportation supply and travel demand is determined and represented by the flow pattern of the network.

Simulation models and network equilibrium models are two major categories of traffic flow prediction models. Simulation models can be further divided into (1) microscopic models based on individual driving behavior, vehicle characteristics, and the interactions between vehicles in the traffic stream; (2) macroscopic models based on deterministic relationships developed through research on highway capacity and traffic flow theory. In general, simulation models can represent better the evolution of traffic over time; however, route choice behavior is usually not considered in simulation models. Thus, simulation models are not adequate for network-wide traffic flow problems, as compared with the local, small-scale impact evaluations for ATMS/ATIS.

Network equilibrium models (also known as traffic assignment models) are computational procedures for predicting traffic flows on networks based on assumptions about drivers' route choice behavior; thus, these models are suitable for ATMS/ATIS evaluation. Network equilibrium models can be generally categorized into static models and dynamic models. Static models are well established and now widely accepted by transportation planning professionals. However, static models have only limited capabilities to explore traffic dynamics. Moreover, there are inevitable shortcomings inherent in static models and their associated results. Simply put, these shortcomings arise from the fact that static models are based on pre-specified, constant levels of travel demand. The time-independent link flows and link travel times from static models are therefore unable to capture the dynamics of network traffic flows. In view of the goals of ATMS/ATIS, it is clear that conventional static route choice models are not capable of fulfilling the needs for analyzing and evaluating ATMS/ATIS. In contrast, dynamic models based on the time-varying character of travel demand and traffic flows, as well as time-dependent traffic conditions, need to be developed and applied. Considering the needs of ATMS/ATIS, dynamic models are more appropriate prediction and evaluation tools for urban transportation networks.

Generally, dynamic network equilibrium models are studied mainly through the following approaches: (1) optimization approach: the pioneering effort in this field was by Merchant and Nemhauser (1978a, 1978b), Carey (1986, 1987, 1992), Janson (1991a, 1991b, 1992) and Janson and Robles (1995); (2) optimal control approach: the pioneering effort was by Luque and Friesz (1980); later Friesz et al. (1989, 1993), Wie (1989, 1991), Wie et al. (1990), and Ran et al. (1992, 1993, 1994) proposed various optimal control theory-based dynamic route choice models; (3) simulation-assignment approach: two recognized models adopting this approach are INTEGRATION (Van Aerde and Yager, 1988) and DYNASMART (Mahmassani and Peeta, 1993).

Compared with optimization and optimal control approaches, simulation-assignment models usually lack an analytical model formulation. Proofs of existence, uniqueness and convergence of solutions are difficult to obtain for simulation-assignment models. Simulation-assignment models, however, may be easier to incorporate into traffic control schemes which are applied in ATMS/ATIS. Optimal control theory-based models exhibit the attractiveness of sound theoretical formulations and provide continuous-time formulations which are able to reach the defined equilibrium states. However, computationally practicable procedures for solving large-scale control theory-based route choice models have not been developed as yet.

This paper describes a large-scale dynamic route choice model being developed as a means of predicting traffic flows for ATMS and ATIS. The adopted dynamic route choice model, DYMOD, was originally formulated by Janson (1991a). Janson's dynamic algorithm is employed to solve the model to convergence. For detailed descriptions of the model formulation, see Janson (1991a) and Janson and Robles (1993, 1995). One of the major goals of this research is to generate time-dependent traffic characteristics for a large network. To this end realistic traffic engineering-based link delay functions, such as Akcelik (1988) functions, are applied for better estimations of link delays at various types of links and intersections.

Section 2 describes the solution algorithm of the adopted model. The link travel time functions being applied are discussed in Section 3. The ADVANCE network representation, travel demand and input data preparation are presented in Section 4. Computational results and analyses are reported in Section 5. Conclusions and a summary are given in the last section.

2 Solving the Model

The solution algorithm for DYMOD can be intuitively described in two steps. The first step maintains the temporally-correct and time-continuous traffic flow propagation by solving a shortest route problem with time-dependent flows and first-in-first-out (FIFO) constraints. The shortest route algorithm used in this research has been modified for dynamic problems. Link travel times used in pivoting from a node depend on the time interval in which the shortest route tree departs from that node and the link flow in that time interval. Then a multi-interval, time-varying-demand route choice problem is solved for these flows and link times. This step can be viewed as solving a sequence of static route choice problems. Static route choice problems can be solved quite efficiently by convex combination methods (e.g., the Frank-Wolfe (F-W) algorithm and PARTAN) for nonlinear programs with linear constraints. These methods may create temporally discontinuous flows when directly applied to dynamic route choice problems.

Applying this procedure iteratively, an approximate dynamic user-optimal state is obtained after several iterations. The algorithm terminates when the similarity of the dynamic user-optimal state obtained from the above two steps are within a prespecified tolerance. The steps of Janson's dynamic algorithm is shown in Figure 1.

Adjustments of link capacities are made between the two steps to account for capacity changes caused exogenously (e.g., signal timing changes, incidents, and other unexpected capacity reduction events) or generated endogenously (e.g., queue spillbacks). Adjustments of link capacities are made on inflow links to nodes whose outflow links have flow greater than γ times of the original capacity ($\gamma = 1.05$ is being used). If queues on outflow links have spilled back to their tail node (at the intersection), then speeds on inflow links should approximate the weighted speed of outflow links.

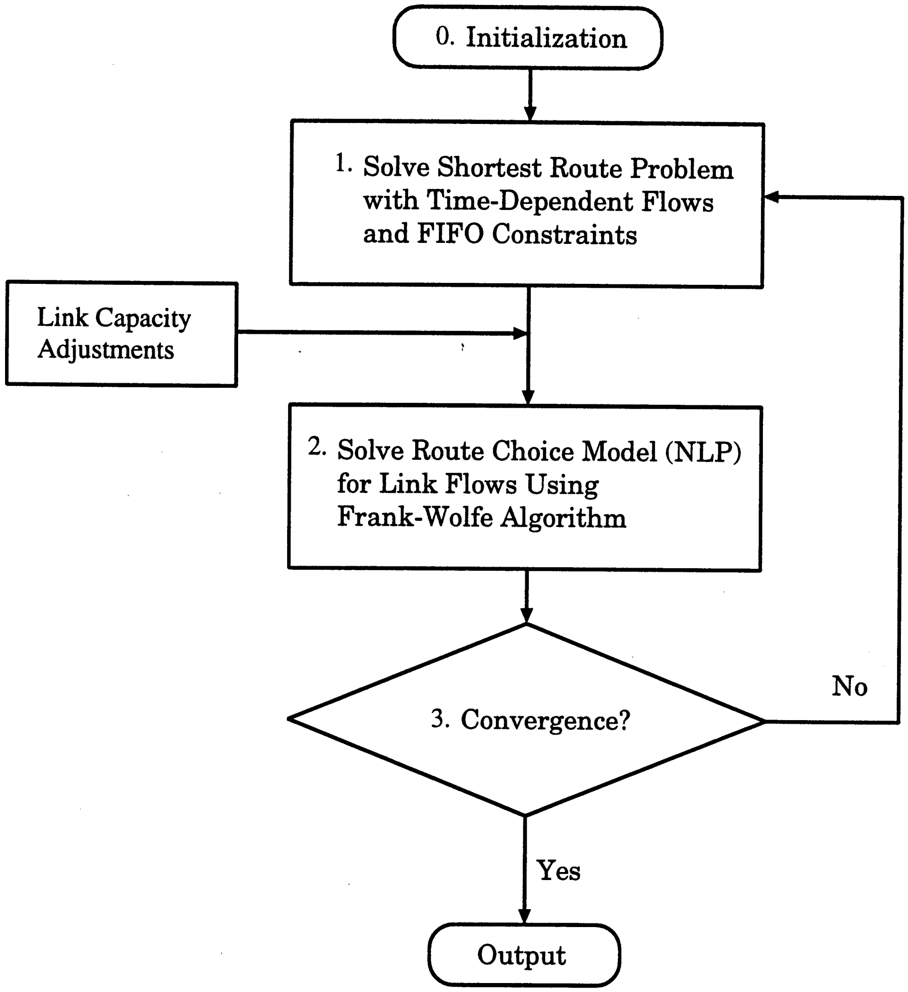

Figure 1: Flowchart of Janson's DYMOD Algorithm

The fraction of link k that is blocked by a queue within time interval t, (π_k^t), must also be measured. If π_k^t is smaller than 1, then there is no effect of inflows to link k in time interval t. If π_k^t is greater than 1, then the portion of the time interval of the inflow link affected by this queue is determined. Meanwhile, whether the queue is decreasing or increasing within the time interval is also decided. The essential steps of these capacity adjustments and queue propagations are:

1. track the location of multiple queue spillbacks in the network;

2. weight the effects of multiple queue spillbacks that jointly affect inflows to any node;

3. adjust the capacities of the inflow links to each node in proportion to the fractions of

flows affected by each queue.

The procedure terminates when all nodes in all time intervals have been processed and the change of capacity of any link is less than γ times of the original capacity. Robles and Janson (1995) compares the results of DYMOD to traffic surveillance data from I–25 in Denver, and concludes the performance of the model is excellent.

3 Link Travel Time Functions

This section presents mathematical functions used within the route choice model to estimate link travel times for given flow rates. The choice of the delay functions involves several criteria: (1) the desired mathematical properties of the function to satisfy the condition for a unique solution of the model; (2) the cost and limited availability of road data; (3) the computational effort required by the model; and (4) the desired accuracy of the travel time estimates generated by the model.

One of the goals of this research is to model time-dependent travel times by turning movement. Analytical functions are preferred over regression-based models because the former generate reasonable estimates over a much wider range of input flows and other parameters. Criteria (1) and (2) above exclude many highly detailed traffic engineering-based models. Criterion (3) excludes simulation models, which are suitable only for small networks.

Delay functions selected for this study can be classified by road type and intersection type. First, delay functions for signalized intersections are presented (Section 3.1), next for unsignalized intersections (Section 3.2), and then for freeway-related facilities (Section 3.3).

3.1 Signalized Intersections

The specific delay function for links at signalized intersections applied in the study has the following form (Akcelik, 1988):

$$d = \frac{0.5C(1-u)^2}{1-ux} + 900T\gamma \left[x - 1 + \sqrt{(x-1)^2 + \frac{8(x-0.5)}{cT}} \right] \tag{1}$$

where d is the average delay per vehicle (second/vehicle), C is the signal cycle length (seconds), $u = g/C$ is the green split, g is the green time (seconds), x is the flow-to-capacity ratio, T is the duration of the flow (hour). The parameter $\gamma = 1$ for $x > 0.5$ and 0 otherwise. Figure 2 shows an example of an Akcelik function.

The first term, called the *uniform delay*, was originally developed by Webster (1958). It reflects the average delay experienced by drivers in *undersaturation conditions*, that is when the arrival flow does not exceed capacity. In oversaturation conditions, $x = 1$ is used in the uniform delay term. The second term of Equation (1) is called the *overflow delay.* It reflects the delay experienced by the vehicles when the flow rate is close to or exceeds capacity. Temporary overflow at an intersection may also occur when the average arrival rate is lower than the capacity, due to a random character of the arrival pattern. The earliest delay functions (for example, Webster, 1958) were based on the steady-state model and were defined only for undersaturation conditions.

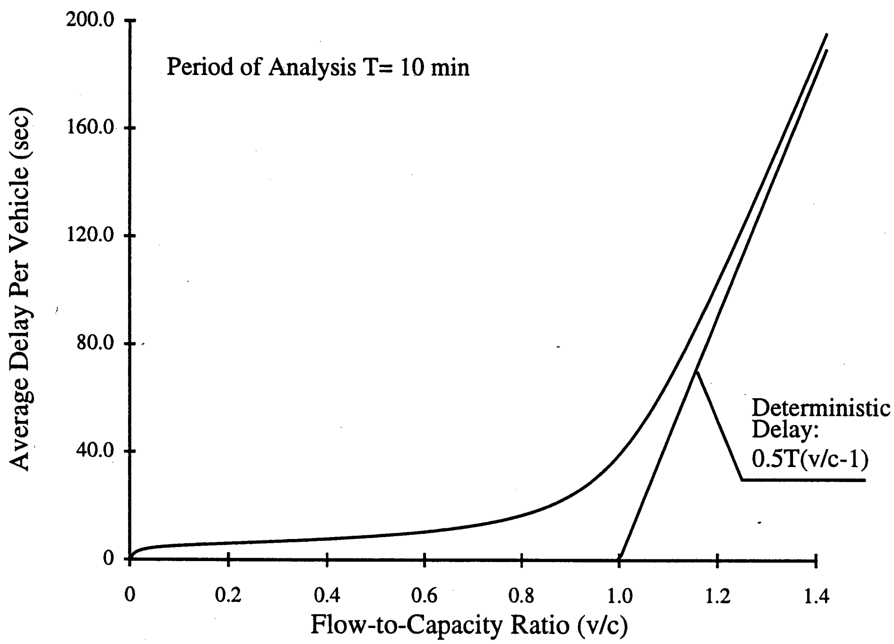

Figure 2: Steady-State Delay Model vs. Time Dependent Formulae

3.2 Unsignalized Intersections

Delay functions for unsignalized intersections, whether major/minor priority intersections or all-way-stop controlled intersections, are discussed next. For major/minor priority intersections, formulae developed by Kimber and Hollis (1979) are adopted by a related study of an asymmetric route choice model. While the delay function developed by Kimber and Hollis is being tested by that study in the large-scale network, the BPR function is temporarily used for estimating delays at major/minor priority intersections.

As for the delay function for all-way-stop intersections, the following exponential delay model is used (Meneguzzer, 1995):

$$d = \exp\left[3.802(v/c)\right] \tag{2}$$

where d is the average approach delay, v is the total approach flow, and c is the approach capacity. Note that the form of this exponential function, which is relatively flat at low volume-to-capacity ratio but becomes very steep as the degree of saturation increases, reflects the operational characteristics of all-way-stop intersections well. Kytes and Marek (1989) found that approach delay is approximately constant and in a range of five to ten seconds per vehicle for approach flows up to 300 to 400 vehicles/hour, but increases exponentially beyond this threshold. An increase in conflicting and opposing flows has the effect of reducing this threshold. In addition, it should be noted that Equation (2) is suitable for use in a network equilibrium model, since it is defined for any volume-to-capacity ratio.

3.3 Freeway-Related Facilities

Several types of freeway-related facilities occur within the test area: basic freeway segments, ramps and ramp-freeway junctions, weaving sections and toll plazas. A sophisticated scheme of delay functions has been developed for individual freeway-related facilities (Berka et al., 1994). Those functions are expected to be incorporated into the solution procedure in the future. Currently, the BPR function is adopted for freeway-related facilities.

4 The ADVANCE Network and Data Preparation

ADVANCE (Advanced Driver and Vehicle Advisory Navigation Concept), a field test of ATIS was recently concluded by the Illinois Department of Transportation (IDOT) and the Federal Highway Administration (FHWA), in collaboration with the University of Illinois at Chicago, Northwestern University, and the IVHS Strategic Business Unit of Motorola, Inc.

4.1 Test Network

The ADVANCE Test Area is depicted in Figure 3. It is located in the northwestern suburbs of the Chicago area and covers about 300 square miles (770 square kilometers). Dense residential communities, office centers, regional shopping centers, subregional government centers, and the O'Hare International Airport are located in the ADVANCE Test Area. The network topology of the test area is almost a regular grid with a few diagonal major arterials directed towards the Chicago CBD. The freeway system includes I–90, I–94, I–190, I–290, I–294, IL–20 and IL–53. Except for the remote northwest corner, the freeways serve nearly all parts of the Test Area. The southwest quadrant is characterized by modern, multi-lane arterials designed for high volumes. In Figure 3, collectors, arterials and freeways are drawn with lines of different widths, freeways being the widest line. The heavy black line indicates the boundary of the ADVANCE Test Area.

4.2 Travel Demand

The ADVANCE Test Area is divided into 447 zones, originally specified by the Chicago Area Transportation Study (CATS), to assign time-dependent travel demand. Actually, these zones define a somewhat larger area called the extended test area, which is used to perform the assignment. Daily trip tables based on CATS estimates for 1990 were factored to represent travel demand for five time-of-day periods (night, 12 am to 6 am; morning peak, 6 am to 9 am; mid-day, 9 am to 4 pm; afternoon peak, 4 pm to 6 pm; and evening, 6 pm to 12 am). Each time-of-day period is divided into 10-minute intervals used for solving the dynamic route choice model. The 10-minute departure rates for each origin zone are derived

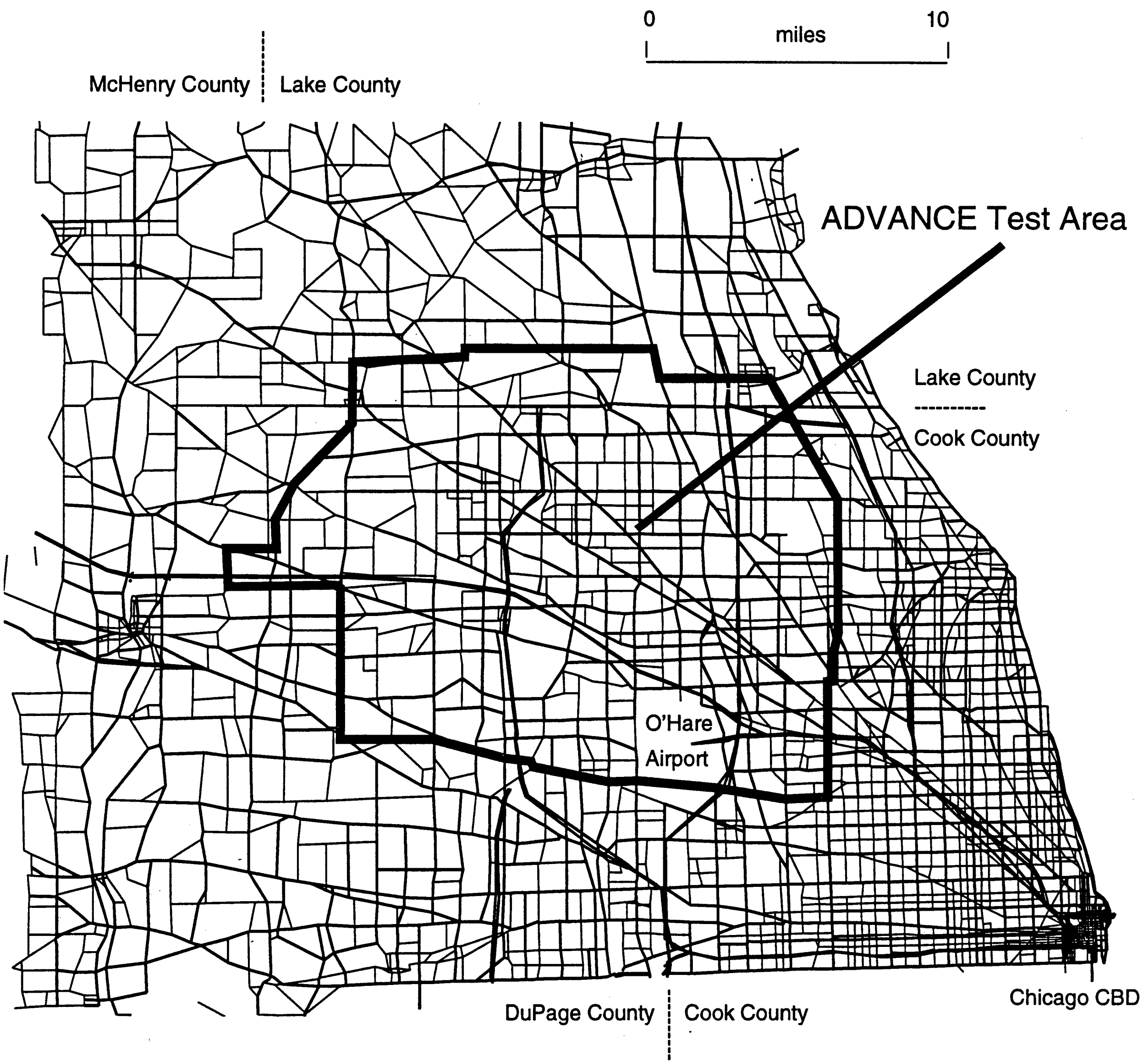

Figure 3: Test Area and Network in Northwestern Suburbs of Chicago

from the time-of-day half-hour departure rates obtained from CATS. Trips originating and/or terminating outside the Test Area are represented by zones on the boundary. An auxiliary analysis of these flows is based on route flows generated by a static route choice model for a larger region encompassing the ADVANCE Test Area (Zhang et al., 1994).

Table 1: Network Characteristics

Type of Facility	No. of Links
Arterial/Collector	4061
Tollway/Freeway	197
Freeway Ramp	202
Toll Plaza	14
Freeway Weaving Section	11
Centroid Connector Links	2491
Approach Link	874
Total	7850
Number of Nodes	2552
Number of Zones	447

Table 2: Intersection Frequency by Number of Legs and Control Type

	Signalized	Priority	All-way-stop	Total
Three-leg	257	174	51	482
Four-leg	558	52	60	670
Five-leg and more	7	0	0	7
Total	822	226	111	1159

4.3 Expanded Network Representation

In a conventional route choice model, the network is coded so each intersection is represented as a single node and each approach is represented as a single link. In the conventional network representation, 7,850 approach links and 2,552 nodes are included in the ADVANCE Test Area (Table 1). Table 1 also lists the frequency of links by the facility type. Table 2 presents the breakdown of arterial/collector intersections by the number of the legs and control type. To account for better link flow/delay relationships and potential queue spillback effects, each turning movement is coded as a separate link called an intersection link. For example, a typical four-leg intersection with two-way approaches without any turning restrictions (U-turn excluded), four approach nodes, four exit nodes and twelve intersection links are required in this expanded network representation (see Figure 4).

The expanded intersection representation procedure is applied only to nodes representing

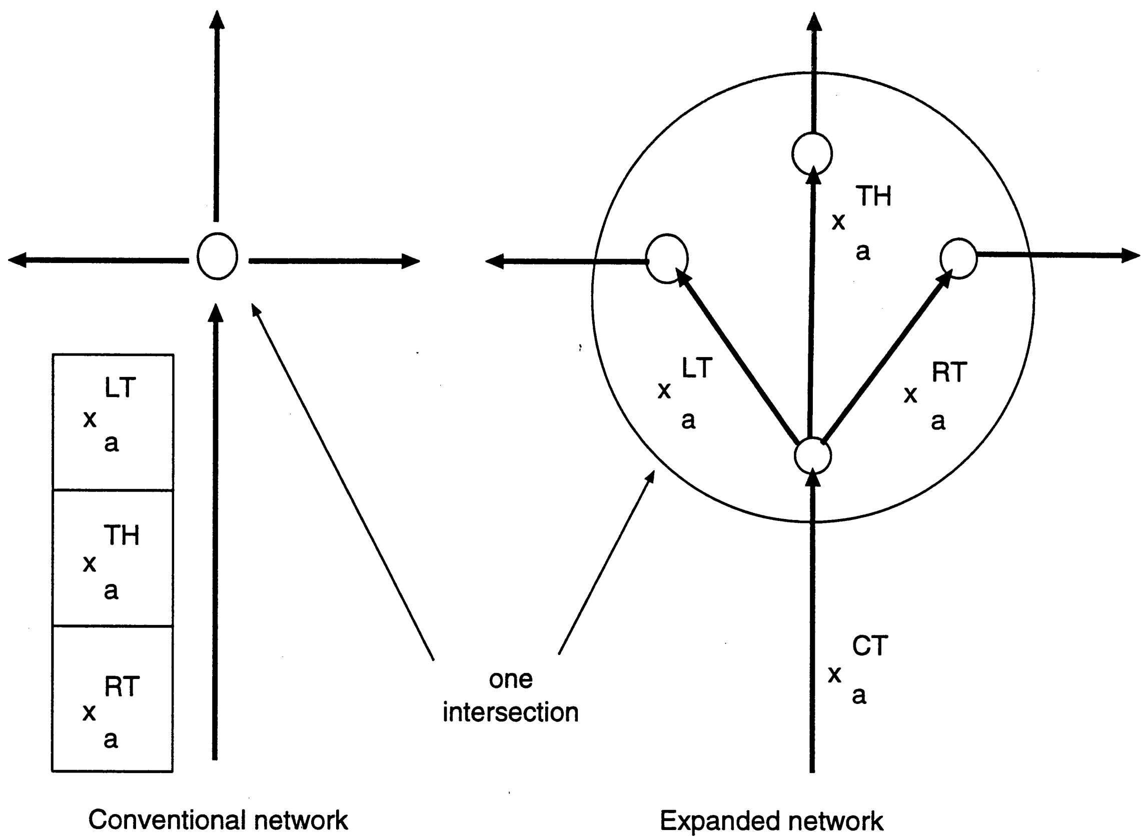

Figure 4: Expanded Intersection Representation

an intersection of arterials or collectors. Nodes that do not need to be expanded are freeway nodes, no-delay intersections, and other nodes not representing intersections. Because of the expanded network representation, the network size increases about three times in comparison with conventional network representation. To that end, 22,918 links and 9,700 nodes are actually modeled for solving the dynamic user-optimal route choice model. Note that the detailed delay functions by turning movements are applied only within the actual ADVANCE Test Area. The network expansions are performed by the network builder program developed by Berka et al. (1994) and Meneguzzer (1995).

4.4 Traffic Input Data

In order to utilize traffic engineering-based link delay functions instead of BPR-type functions, many additional data need to be presented. Besides the typical input data such as link capacity and free flow travel time, types of turning movements (left, through, right), link facility types (e.g., centroid connector, freeway, tollway, arterial, etc.), intersection categories, types of traffic control at intersections (e.g., signalized, priority, all-way-stop), cycle length, saturation flow, and signal timing split are required with this regard.

A static asymmetric user-optimal route choice model for the ADVANCE Network was used to generate the above information (Berka et al., 1994). That static asymmetric user-optimal route choice model was the basis for generating the initial static link travel time profiles of the ADVANCE Project.

5 Computational Results and Analyses

The model was implemented on the CONVEX–C3880 at the National Center for Supercomputing Applications (NCSA), University of Illinois at Urbana-Champaign. The Convex–C3880 is a vector shared memory machine consisting of 8 processors (240 MFLOPS per processor peak), 4 Gbytes of memory and 60 Gbytes disk space.

For the ADVANCE Network with morning peak (18 ten–minute intervals) travel demand of nearly 560,000 trips, 432 Mbytes of memory and 55.67 CPU hours are needed to reach a very fine convergence (see definition in the next subsection) from a zero flow initial solution. A modified version of DYMOD can be executed with much less memory (only 55 Mbytes for the same problem size described above) and thus can be implemented on a workstation; however, it requires more CPU time and more disk space.

5.1 Network Performance Measures

Five global network performance measures are defined to monitor the solution process of Janson's dynamic algorithm and to assess traffic conditions over the ADVANCE Network. Definitions of these global network performance measures are listed as below:

1. *Average travel distance*

$$\bar{\ell} = \frac{1}{R} \sum_a \sum_t \ell_a x_a[t]$$

   where $\bar{\ell}$ is the average travel distance (miles), R is the total number of trips per hour (vph), ℓ_a is the length of the link a (miles) and $x_a[t]$ is the flow on the link a during time interval t (vph).

2. *Average travel time*

$$\bar{c} = \frac{1}{R}\sum_a \sum_t c_a[t] x_a[t]$$

where $\bar{c}$ is the average travel time (minutes) and $c_a[t]$ is the travel time on the link a at the time interval t (minutes).

3. *Network space mean speed*

$$\bar{S} = \bar{\ell}/\bar{c}$$

4. *Average flow-to-capacity ratio*

$$\bar{x} = \frac{1}{K}\frac{1}{T}\sum_a \sum_t \frac{x_a[t]}{C_a[t]} x_a[t]$$

where $\bar{x}$ is the average flow-to-capacity ratio, K is the total number of links , T is the number of time intervals and $C_a[t]$ is the capacity of the link a at the time interval t (vph).

5. *Convergence measure*

Define $\alpha_{ri}^{d}[t]$ as a *node time interval*; $\alpha_{ri}^{d}[t] = 1$ indicates that the flow departing zone r in time interval d has crossed node i in time interval t, and 0 otherwise. The changes of *node time intervals* provide an index of the DUO state obtained from the two steps described in the solution algorithm. The solution algorithm will terminate if the changes of node time intervals (NDIFFS) are less than $nodes \times zones \times intervals \times x\%$. This index indicates that only $x\%$ changes of node time intervals are allowed for the last trip departing from each zone over the total analysis period. The decision of the x-value depends on the desired solution accuracy. In the case of perfect convergence, the changes of node time intervals equal to zero.

5.2 Analysis of Results

Table 3 shows selected characteristics of the final solution for the morning peak period of the ADVANCE Network and separate road classes (collector, arterial and freeway-related

Table 3: Characteristics of the Final Solution for AM Peak Period by Road Class

Link Class	Travel Distance (miles)	Travel Time (minutes)	Mean Speed (mph)
Collector	1.22 (1.34)	4.12 (5.97)	17.80 (13.47)
Arterial	6.18 (6.83)	16.14 (21.65)	22.97 (18.92)
Freeway	2.63 (3.01)	3.29 (3.95)	48.02 (45.73)
All Classes	10.03 (11.18)	24.19 (31.61)	24.89 (21.22)

(·) results from Berka et al. (1994)

facilities). Figures 5, 6, 7 and 8 show the variations of the network performance measures for solutions of Step 1 of the algorithm.

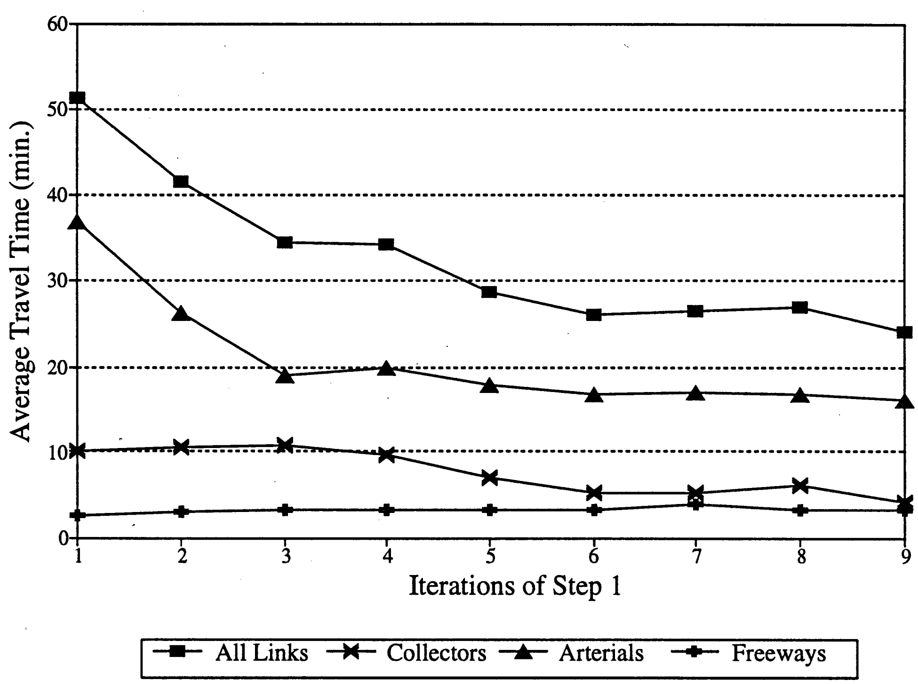

Figure 5: Average Travel Time

For the convergence measure, $nodes = 9,700$, $zones = 447$, $intervals = 18$ and $x = 0.1$ were used in this solution. Note a rather small value was chosen for x indicating a fine convergence of the algorithm is desired. After 9 outer iterations of Step 1, NDIFFS equals to $55,741$ showing the algorithm has converged at the level of $x \approx 0.07$. The rate of change of

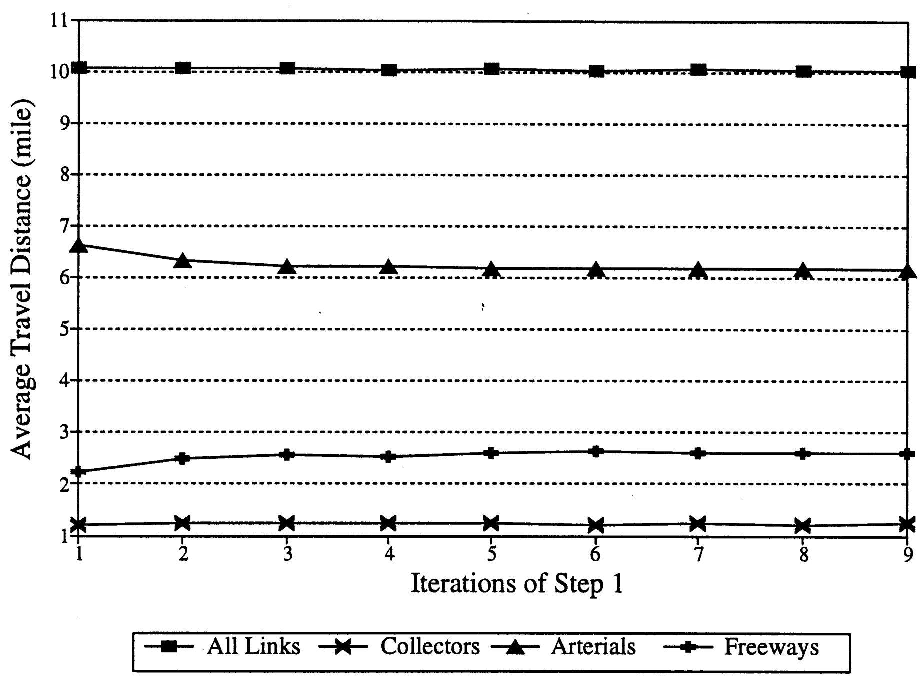

Figure 6: Average Travel Distance

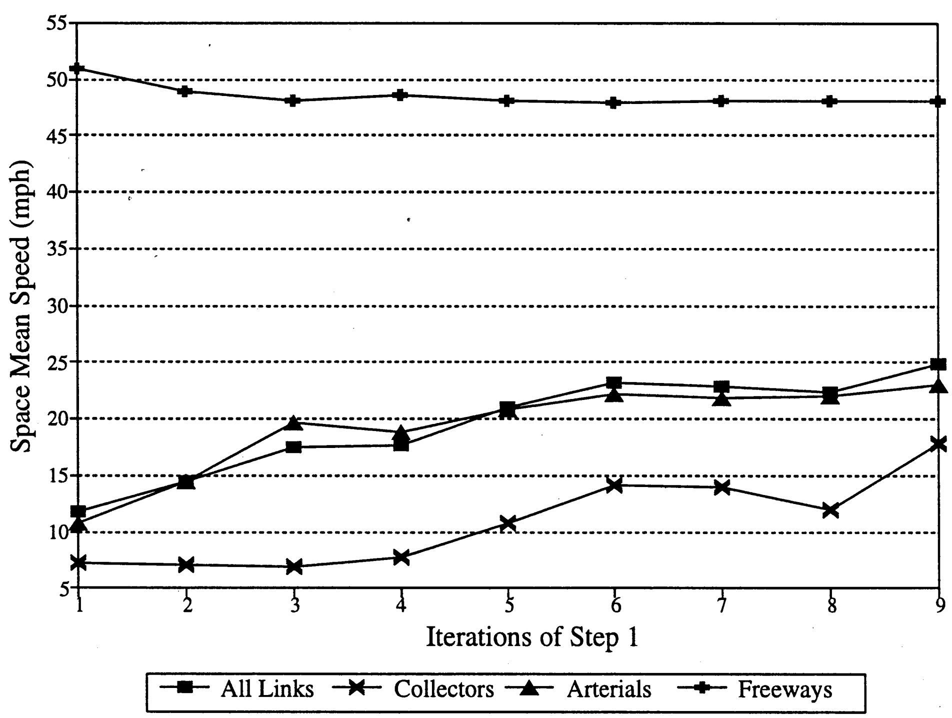

Figure 7: Network Space Mean Speed

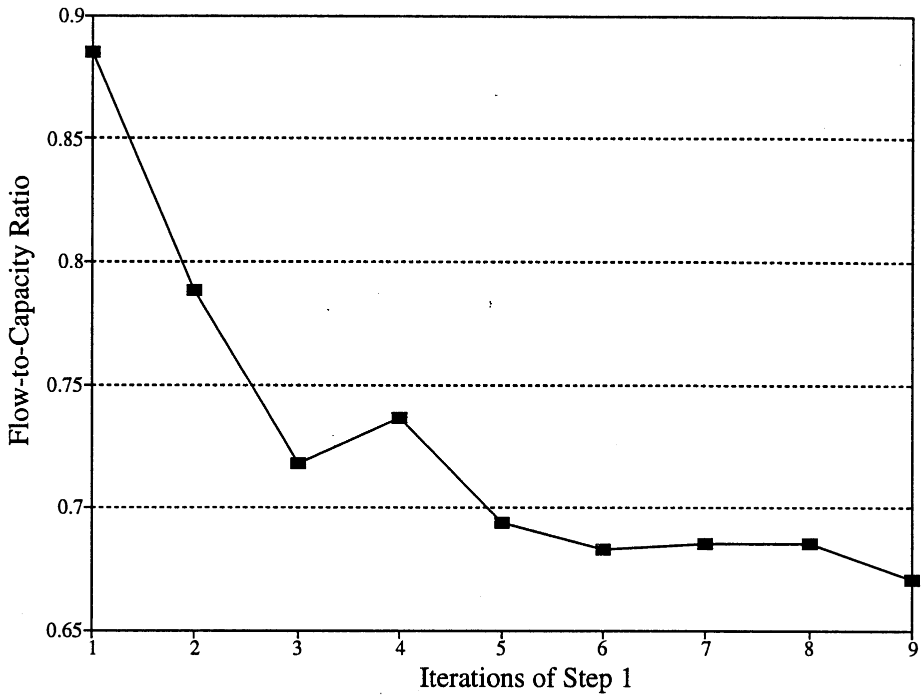

Figure 8: Flow-to-Capacity Ratio

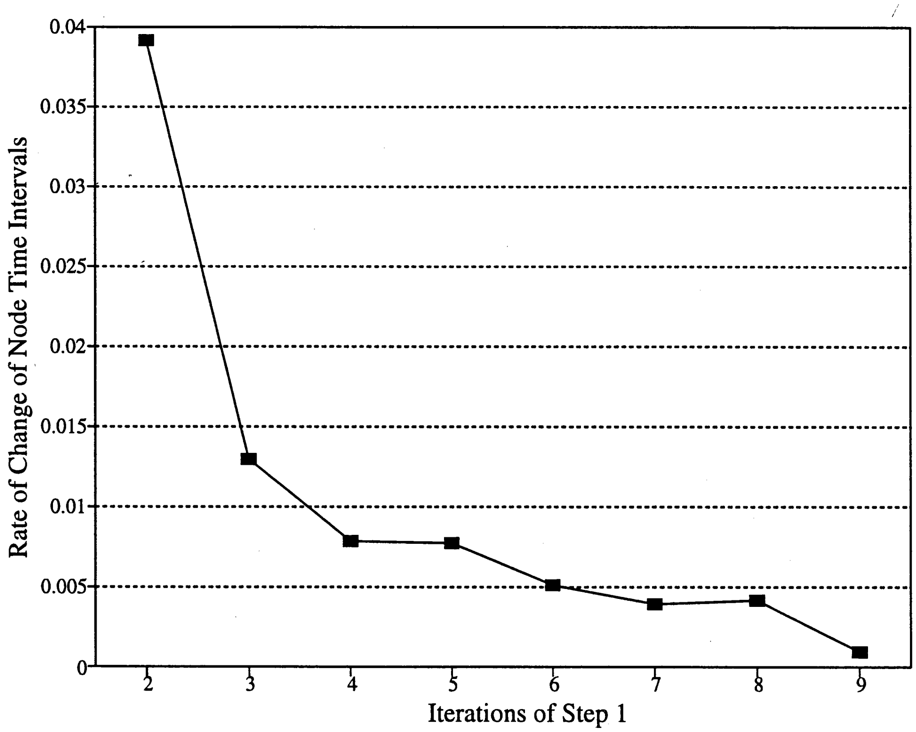

Figure 9: Rate of Change of Node Time Intervals

node time intervals is displayed in Figure 9 which indicates that this model was solved quite smoothly by the solution algorithm for the ADVANCE Network. The flow-to-capacity ratio was 0.67 using this dynamic model versus 0.78 using the asymmetric static model (Berka et al., 1994).

Unfortunately, link flows and link travel time data are not available for the ADVANCE Network, either in general, or more specifically for the O-D matrix used in this solution. These data, as well as route flow data, are urgently needed to advance the state of the art of network modeling for ITS.

6 Summary and Conclusions

This paper presented a dynamic user-optimal route choice model called DYMOD for predicting real-time traffic flows for Advanced Traffic Management Systems (ATMS) and Advanced Traveler Information Systems (ATIS) such as the ADVANCE Project. Model refinements, extensions and computational results from the ADVANCE Network were presented. Realistic traffic engineering-based link delay functions were adopted in our study for better estimations of traffic dynamics. Using Janson's algorithm, the model has been solved, convergence and various sets of predicted time-dependent traffic characteristics have been obtained and analyzed. We believe this is the largest dynamic route choice model which has been solved thus far. The proposed model is being extended to be capable of modeling the effects of enroute diversions caused by highway incidents such as stalled vehicles, dropped objects and traffic accidents, etc.

To date, very few traffic flow prediction models are suitable for ATMS and ATIS applications. Although not yet fully validated, DYMOD is able to predict time-dependent traffic characteristics for a large-scale traffic network which are reasonable and internally consistent. Eventually, dynamic route choice models should be integrated into a traffic control and management center to support the decisions on the adjustments of arterial signal timing, ramp metering, incident management and future route guidance strategies, etc.

Acknowledgements

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References

Akcelik, R. (1988) Capacity of a Shared Lane, *Australian Road Research Board Proceedings*, **14(2)**, 228–241.

Berka, S., Boyce, D.E., Raj, J., Ran, B. and Zhang, Y. (1994) *A Large-Scale Route Choice Model with Realistic Link Delay Functions for Generating Highway Travel Times*, Report to Illinois Department of Transportation, Urban Transportation Center, University of Illinois, Chicago.

Carey, M. (1986) A Constraint Qualification for a Dynamic Traffic Assignment Model, *Transportation Science*, **20**, 55–88.

Carey, M. (1987) Optimal Time-Varying Flows on Congested Networks, *Operations Research*, **35**, 58–69.

Carey, M. (1992) Nonconvexity of the Dynamic Traffic Assignment Problem, *Transportation Research*, **26B**, 127–133.

Friesz, T.L., Luque, F.J., Tobin, R.L. and Wie, B.-W. (1989) Dynamic Network Traffic Assignment Considered a Continuous Time Optimal Control Problem, *Operations Research*, **37**, 893-901.

Friesz, T.L., Bernstein, D., Smith, T.E., Tobin, R.L. and Wie, B.-W. (1993) A Variational Inequality Formulation of the Dynamic Network User-Equilibrium Problem, *Operations Research*, **41**, 179–191.

Janson, B.N. (1991a) Dynamic Traffic Assignment for Urban Networks, *Transportation Re-*

search, **25B**, 143–161.

Janson, B.N. (1991b) A Convergent Algorithm for Dynamic Traffic Assignment, *Transportation Research Record*, **1328**, 69–80.

Janson, B.N. (1992) A General Dynamic Assignment Model and Solution Algorithm, Paper presented at the *39th North American Meeting of the Regional Science Association International*, Chicago.

Janson, B.N. (1995) Network Design Effects of Dynamic Traffic Assignment, *Journal of Transportation Engineering/ASCE*, **121**, 1–13.

Janson, B.N. and Robles, J. (1993) Dynamic Traffic Assignment with Arrival Time Costs, *Transportation and Traffic Theory*, 127–146, Elsevier Science Publishers, Amsterdam.

Janson, B.N. and Robles, J. (1995) A Quasi-Continuous Dynamic Traffic Assignment Model, *Transportation Research Record***1493**, 199–206.

Kimber, R.M., and Hollis, E.M. (1979) Traffic Queues and Delays at Road Junctions, *Transport and Road Research Laboratory Report*, **909**, Crowthorne, Berkshire.

Kyte, M. (1991) Interim Materials on Unsignalized Intersection Capacity, *Transportation Research Circular* **373**, 10-35.

Luque, F.J. and Friesz, T.L. (1980) Dynamic Traffic Assignment Considered as a Continuous Time Optimal Control Problem, Paper presented at the *TIMS/ORSA Joint National Meeting*, Washington, D.C.

Mahmassani H.S. and Peeta, S. (1993) Network Performance under System Optimal and User Equilibrium Dynamic Assignments: Implications for ATIS, *Transportation Research Record*, **1408**, 83–93.

Meneguzzer, C. (1995) An Equilibrium Route Choice Model with Explicit Treatment of the Effect of Intersection, *Transportation Research*, **29B**, 329–356.

Merchant, D.K. and Nemhauser, G.L. (1978a) A Model and an Algorithm for the Dynamic Assignment Problems, *Transportation Science*, **12**, 183–199.

Merchant, D.K. and Nemhauser, G.L. (1978b) Optimality Conditions for a Dynamic Traffic Assignment Model, *Transportation Science*, **12**, 200–207.

Ran, B., Boyce, D.E., and LeBlanc, L.J. (1992) Dynamic user-optimal departure time and route choice model: A bilevel optimal-control formulation, Paper presented at *TIMS/ORSA Joint National Meeting*, Orlando.

Ran, B., Boyce, D.E. and LeBlanc, L.J. (1993) A New Class of Instantaneous Dynamic User-Optimal Traffic Assignment Models, *Operations Research*, **41**, 192–202.

Ran, B. and Boyce, D.E. (1994) *Dynamic Urban Transportation Network Models*, Springer-Verlag, New York.

Robles, J. and Janson, B.N. (1995) Dynamic Traffic Modeling of the I–25 HOV Corridor Southeast of Denver, *Transportation Research Record*, **1516**, 48–60.

Van Aerde, M. and Yager, S. (1988) Modeling Integrated Freeway/Traffic Signal Networks: A Proposed Routing-Based Approach, *Transportation Research*, **22A**, 445–453.

Webster, F.V. (1958) Traffic Signal Settings, *Road Research Technical Paper*, **39**, Road Research Laboratory, Her Majesty's Stationery Office, London.

Wie, B.-W. (1989) An Application of Optimal Control Theory to Dynamic User-Equilibrium Traffic Assignment, *Transportation Research Record*, **1251**, 66–73.

Wie, B.-W. (1991) Dynamic Analysis of User Optimized Network Flows with Elastic Demand, *Transportation Research Record*, **1328**, 81-87.

Wie, B.-W., Friesz, T.L. and Tobin, R.L. (1990) Dynamic User-Optimal Traffic Assignment on Congested Multi-Destination Networks, *Transportation Research*, **24B**, 431–442.

Zhang, Y., Hicks, J. and Boyce, D.E. (1994) Trip Data Fusion in ADVANCE, *ADVANCE Working Paper* No. **43**, Urban Transportation Center, University of Illinois, Chicago.