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Technical Report Number 59
February, 1997

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A Variational Inequality Model of an Ideal Dynamic User-Optimal Route Choice Problem[1]

David E. Boyce[2], Der-Horng Lee[2,3] and Bruce N. Janson[4]

Abstract

This paper presents an ideal dynamic user-optimal (DUO) route choice model for predicting dynamic traffic conditions, intended for off-line Advanced Traffic Management Systems (ATMS) and Advanced Traveler Information Systems (ATIS) evaluation and implementation. The proposed ideal DUO route choice model is formulated as a variational inequality model. Variational inequality (VI) models provide the most generalized formulation for describing a dynamic network equilibrium. Although route-based VI models have an intuitive interpretation, their computational complexity makes them intractable for realistic applications because of the route enumeration requirement. Consequently, the proposed ideal DUO route choice model is formulated as a link-based variational inequality for the need of large-scale implementations.

Using the developed diagonalization algorithm with discrete time intervals, this model is solved to convergence. Computational results and analyses from a real and large-scale traffic network, the ADVANCE Network, are presented.

1 Introduction

The *ideal* dynamic user-optimal (DUO) route choice problem is to determine vehicle flows at each instant of time so that the travel times experienced by vehicles departing at the same time and with the same origin-destination (O-D) attributes are minimal and equal. This paper provides a discrete-time formulation and solution algorithm for the ideal DUO route choice problem.

Since variational inequality (VI) models provides the most generalized formulation for describing a dynamic network equilibrium, a link-time-based VI model is proposed. Dynamic network constraints including definitional constraints, flow conservation constraints, flow propagation constraints, first-in-first-out (FIFO) constraints and nonnegativity constraints are presented and discussed in detail. Although the proposed model exhibits a discrete-time formulation, temporally-correct routes and time-continuous flow propagations are maintained in a quasi-continuous manner. Based on the link-time-based ideal DUO route choice conditions, the equivalent variational inequality is derived. Derivation and discussion are presented.

[1]Paper will be published as a book chapter by the *4th Meeting of the EURO Working Group on Transportation*, September 9–11, 1996, Newcastle upon Tyne, U.K.

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A diagonalization algorithm is proposed to solve this VI model using 5-minute time intervals to convergence. First, the *inner* problem which is a multi-interval, time-varying demand route choice problem with fixed *node time intervals* (see Section 3.3 for the definition) is formulated. Then, the shortest route travel times and *node time intervals* constraints are updated in the *outer* problem. The *inner* problem and the *outer* problem are solved iteratively till the convergence is reached.

This model is implemented to the ADVANCE Network which locates in the northwestern suburbs of Chicago area and covers about 800 square kilometers (300 square miles), 447 zones, 406,560 travel flows (4 PM to 6 PM) and nearly 10,000 nodes and 23,000 links in *expanded intersection representation.* To generate time-dependent traffic characteristics for a real network, realistic traffic engineering-based link delay functions such as Akcelik (1988) functions are applied for better estimation of link delays at various types of links and intersections. This is the largest dynamic route choice solution which has been obtained thus far, to the best knowledge of the authors. The model was solved using the CONVEX–C3880 at the National Center for Supercomputing Applications (NCSA), University of Illinois at Urbana-Champaign. Convergence and computational results are obtained and analyzed.

2 Ideal Dynamic User-Optimal State

The travel-time-based ideal DUO state is defined as (Ran and Boyce, 1996):

> **Travel-Time-Based Ideal DUO State:** *For each O-D pair at each interval of time, if the actual travel times experienced by travelers departing at the same time are equal and minimal, the dynamic traffic flow over the network is in a travel-time-based ideal dynamic user-optimal state.*

Under the *ideal* DUO state, travelers have no reason to change their routes. Therefore, the obtained DUO state can be viewed as an equilibrium.

3 Dynamic Network Constraints

We consider a multiple O-D network that is represented by a directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ where $\mathcal{N}$ is the set of nodes and $\mathcal{A}$ is the set of directed links. In the following constraint sets, the index r denotes an origin and s denotes a destination. In addition, both d and t ($t \geq d$) denote a time interval. However, d denotes the departure time interval and t denotes a specific time interval during the journey.

3.1 Definitional Constraints

Consider a fixed time period $[0, \mathcal{T}]$ which is long enough to allow all vehicle flows departing during the peak period to complete their journeys. Let $x_a[t]$ be the total flow on link a in time interval t and $x_a^{rs}[d]$ be the flow of vehicles from origin r to destination s on link a that departed in time interval d.

Therefore, for total flow on link a in time interval t, Equation (1) must hold.

$$x_a[t] = \sum_{d=1}^{t} \sum_{rs} x_a^{rs}[d] \phi_{ri}^d[t] \quad \forall a, t;\ a = (i, j) \tag{1}$$

where $\phi_{ri}^d[t]$ is the fraction of all flows departing zone r in time interval d that crosses node i in time interval t. Equation (1) defines total flow on link a in time interval t to be the sum of flows departing from zone r in any time interval d from interval 1 up to and including t $(t \geq d)$ using link a in time interval t.

3.2 Flow Conservation Constraints

Flow conservation needs to be considered for different types of nodes including intermediate nodes, origins and destinations for a dynamic route choice model. Define π_{ri}^d as the minimal travel time actually experienced by flows departing from origin r to node i in time interval d, where $\bar{\pi}_{ri}^d$ denotes the number of time intervals traversed in π_{ri}^d and Δt is defined as the duration of each time interval.

$$\bar{\pi}_{ri}^d = \omega \quad \text{if} \quad \omega \leq \pi_{ri}^d / \Delta t < \omega + 1 \tag{2}$$

where ω is an integer $(0 \leq \omega \leq T)$. Equation (2) makes the actual travel time π_{ri}^d equal to a multiple of the time increment Δt. Define $q_{rj}[d]$ as flows from zone r to node j departing in time interval d via any route. Let $x_a^{rsp}[t]$ be the flow on link a in time t from zone r to s on route p. Equation (3) constrains the inflow minus outflow at any intermediate node j $(j \neq r, s)$ in each time interval $(t \geq d)$ to the proper departure flows in each time interval between all O-D pairs.

$$\sum_{d=1}^{t} \phi_{rj}^d[t]\ q_{rj}[d] = \sum_{sp} \sum_{d=1}^{t} \left[\sum_{a \in B(j)} x_a^{rsp}[d]\ \phi_{ri}^d[t] - \sum_{a' \in A(j)} x_{a'}^{rsp}[d]\ \phi_{rj}^d[t] \right] \tag{3}$$

$$\forall\, r, j, t;\ a = (i, j);\ a' = (j, k);\ p$$

$$\text{where} \quad \sum_{p} q_{rj}^p[d] = q_{rj}[d] \quad \forall\, r, j, d$$

$A(j)$ is the set of links whose tail node is j, and $B(j)$ is the set of links whose head node is j. Conservation of flow at origin r requires the flow originating at node r in time interval d to equal the flow entering the links leaving origin r in time interval d. Equation (4) states the flow conservation at origins.

$$f_{rs}[d] = \sum_{a \in A(r)} \sum_{p} x_a^{rsp}[d] \qquad \forall\ r, s, d \tag{4}$$

Similarly, conservation of flow at destination s requires the flow exiting at node s in time interval t to equal the flow entering destination s in time interval t. The flow conservation at destinations is expressed by Equation (5).

$$\sum_{a \in B(s)} \sum_{p} x_a^{rsp}[t] = e_{rs}[t] \qquad \forall\, r, s, t \tag{5}$$

Note that $e_{rs}[t]$ is a variable; Equation (5) describes the solution of the model, but does not constrain $x_a^{rsp}[t]$ in this version.

3.3 Flow Propagation Constraints

The proposed DUO route choice model requires nonlinear mixed-integer constraints with *node time intervals* ($\alpha_{ri}^d[t]$) and *flow fractions* ($\phi_{ri}^d[t]$) indicating the time intervals where flows originating from each origin cross each node in order to maintain temporally-correct routes and time-continuous flow propagations over time intervals. Define $\alpha_{ri}^d[t]$ as a [0,1] variable indicating whether the flow departing zone r in time interval d has crossed node i in time interval t. Each *node time interval* acts as an (*if-then*) operator to activate or deactivate certain constraints. A *node time interval* only applies to the last trip (strictly, the end of the platoon or pulse of flow) departing in each departure time interval. The difference between the crossing times at node i of the flows departing in successive time intervals, defined as $\Delta\pi_{ri}^d$, is applied to equations (6)–(8) to determine the temporal spread of trips crossing node i from the same origin.

$$\phi_{ri}^d[t-k] = \left\{\min\left[1, \left(\pi_{ri}^d - (t-1)\Delta t\right)/\Delta\pi_{ri}^d\right]\right\}\alpha_{ri}^d[t] \qquad \forall\, k=0 \tag{6}$$

$$\phi_{ri}^d[t-k] = \left(\Delta t/\Delta\pi_{ri}^d\right)\alpha_{ri}^d[t] \;\; \forall\, k>0 \;\text{ for which }\; \pi_{ri}^{d-1} - (t-1-k)\Delta t \le 0 \tag{7}$$

$$\phi_{ri}^d[t-k] = \left\{\max\left[0, \left(\Delta t(t-k) - \pi_{ri}^{d-1}\right)/\Delta\pi_{ri}^d\right]\right\}\alpha_{ri}^d[t] \tag{8}$$

$$\text{for minimal } k \text{ for which } \pi_{ri}^{d-1}(t-1-k)\Delta t > 0$$

$$\text{where } \;\Delta\pi_{ri}^d = \pi_{ri}^d - \pi_{ri}^{d-1} \;\text{ and }\; \pi_{ri}^0 = \pi_{ri}^1 - \Delta t \;\; \forall\, r,i,d$$

where k is used to count the number of boundaries of time intervals spanned by the difference in node crossing times of the last vehicle in the platoon between successive time intervals.

We now use Figure 1 to explain Equations (6)–(8). In Figure 1, the y-axis denotes the departure time intervals of platoons and the x-axis denotes the sequence of node along an example route. As shown in Figure 1, platoon 1 departs a given origin in its departure time interval 1; the last vehicle of platoon 1 crosses node A in time interval 3. Similarly, Platoon 2 departs the same origin as platoon 1 in time interval 2 and the last vehicle of platoon 2 crosses node A in time interval 4.

Now look at the node crossing times of those two platoons at node D. Equation (6) determines the fraction of platoon 2 crossing node D in time interval 7. This fraction equals the elapsed time between the starting time of interval 7 and the node D crossing time of the end of platoon 2 (represented by dash line in Figure 1), divided by the elapsed time between the node D crossing times of platoons 1 and 2. Equation (6) is designed for $k = 0$. The need to take min[1, etc.] is that this calculation can exceed 1 when computing this fraction for platoons departing in time interval 1, since there is no node crossing time for a previous platoon.

Equation (7) determines the fraction of platoon 2 crossing node D in the *whole* time interval (if any) between the node D crossing times of platoons 1 and 2. The *whole* time interval is interpreted as follows. Suppose platoon 1 crosses node D in time interval 5 and

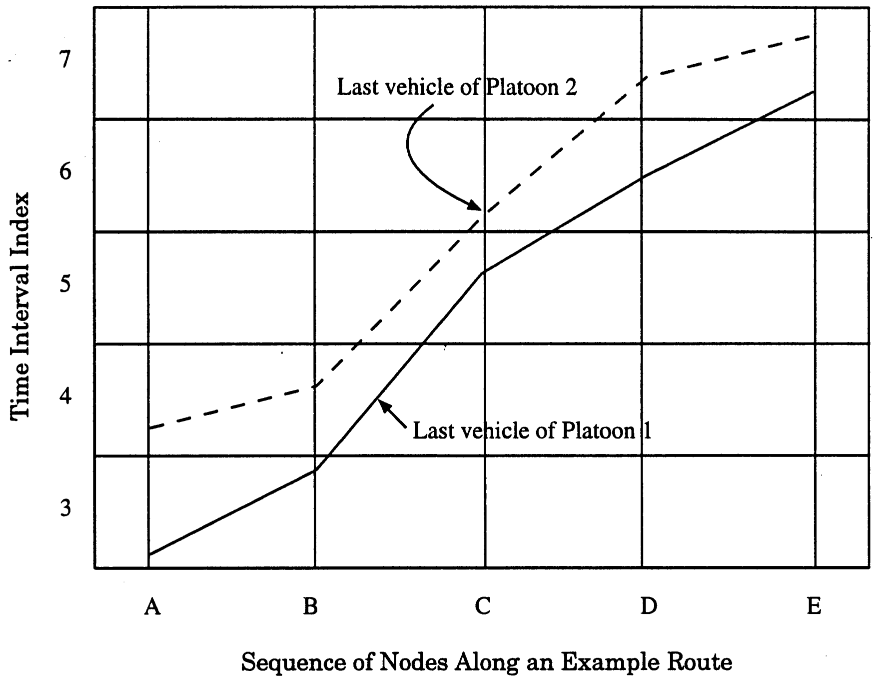

Figure 1: Effect of Flow Propagation Constraints

platoon 2 crosses node D in interval 7 because delays have caused vehicles in platoon 2 to fall farther behind platoon 1. Under this situation, a fraction of platoon 2 crosses node D in interval 5, another fraction of platoon 2 crosses node D with *whole* time interval 6 and another fraction of platoon 2 crosses node D in interval 7. In this case, however, there are no so-called *whole* time intervals.

Equation (8) determines the fraction of platoon 2 crossing node D in time interval 6. This fraction equals the elapsed time between the crossing time of node D of the last vehicle in platoon 1 (represented by solid line) and the starting time of interval 7, divided by the elapsed time between the crossing times of node D of platoons 1 and 2. Equation (6) is designed for $k = 1$. The need to take max[0, etc.] in Equation (8) is that this fraction can be negative when computing this fraction for platoons departing in time interval 1, since there is no node crossing time for a previous platoon.

3.4 First-In-First-Out Constraints

Equations (9)–(12) state FIFO constraints between all O-D pairs according to their travel times in successive time intervals. Define β_{ri}^{d} as the time at which the last flow departing zone r in time interval d crosses node i via its shortest route less FIFO delay time at node i; $\theta_{ri}^{d}[t]$ is defined as the fraction of a time interval t that the last flow departing zone r in time interval d crosses node i; and $\mu_{ra}^{d}[t]$ is defined as the average travel time on link a of the last flow departing zone r in time interval d. The value h $(0 \leq h \leq 1)$ is the fraction of a time interval that the end of the platoon (the last vehicle) departing from zone r in time interval d must follow the end of the platoon departing from zone r in time interval

$d-1$. Vehicles are assumed to make one-for-one (or zero-sum) exchanges of traffic positions along any link, which is an acceptable and expected feature for any aggregate traffic model (Janson and Robles, 1995).

$$\pi^d_{ri} = \max\left(\beta^d_{ri}\ ,\ \pi^{d-1}_{ri} + h\Delta t\right) \quad \forall\, r, i, d \text{ and } \pi^0_{ri} = \pi^1_{ri} - \Delta t \tag{9}$$

$$\theta^d_{ri}[t] = \left[\left(\pi^d_{ri} - (t-1)\Delta t\right)/\Delta t\right]\alpha^d_{ri}[t] \quad \forall\, r, i, d, t \tag{10}$$

$$\mu^d_{ra}[t] = \left[\theta^d_{ri}[t]\tau_a(x_a[t]) + (1-\theta^d_{ri}[t])\tau_a(x_a[s])\right]\alpha^d_{ri}[t] \quad \forall\, r, a, d, t,\ s = t-1 \tag{11}$$

$$\left\{\beta^d_{rj} - \max\left[\pi^d_{ri}\ ,\ (t-1)\Delta t + \Delta\tau^t_a[s]\right]\right\}\alpha^d_{ri}[t] \leq \mu^d_{ra}[t]\alpha^d_{ri}[t] \tag{12}$$

$$\forall\, r, a, d, t,\ s = t-1; \quad \text{where } \Delta\tau^t_a[s] = \tau_a(x_a[s]) - \mu^d_{ra}[t]$$

Equation (9) is a vehicle following constraint that regulates flows departing from the same zone in successive time intervals from passing each other. When solving for π^d_{ri} on the left hand side of Equation (9), π^{d-1}_{ri} on the right hand side is held fixed. If $h = 0$, a trailing platoon can completely overlay (but not overtake) a leading platoon so that the two platoons become coincident, which is not realistic. If $h = 1$, a trailing platoon can never partly gain ground on a leading platoon.

Since Equation (9) does not insure FIFO ordering between all O-D pairs, Equations (10)–(12) are required. Equations (10) and (11) determine the average travel time on link a of the end of the platoon departing zone r in time interval d adjusted for the time into interval t versus $t-1$ that the platoon enters the link. Equations (10) and (11) dampen speed transitions between time intervals in a quasi-continuous manner so that vehicle speeds do not abruptly change if flows enter links just seconds before or after a time interval change.

Equation (12) is needed to prevent FIFO violations in cases where link travel times exceed Δt. Equation (12) does not entirely replace the need for Equation (9). Equation (12) allows trips between different O-D pairs to become concurrent while sharing the same route. Equation (9) insures a minimum separation of the last platoon departing from the same zone in successive time intervals. Trips from the same zone bunch together and cause excessively dense flows if Equation (9) is removed.

3.5 Nonnegativity Constraints

Finally, all variables must be nonnegative at all time intervals. We have

$$x^{rs}_a[t] \geq 0 \qquad \forall\, r, s, a, t; \tag{13}$$

$$e_{rs}[t] \geq 0 \qquad \forall\, r, s, t; \tag{14}$$

$$\phi^d_{ri}[t] \geq 0 \qquad \forall\, r, i, d, t. \tag{15}$$

4 Link-Time-Based Conditions

We now derive the equivalent mathematical inequalities for the travel-time-based *ideal* DUO state using link variables. For any route from origin r to destination s, link a is defined as being used in time interval t if $x^{rs}_a[t] > 0$. Define π^{d*}_{ri} as the minimal travel time actually experienced by flows departing from origin r to node i in time interval d, the asterisk denoting that the travel time is calculated using DUO traffic flows. For link $a = (i,j)$, the minimal travel time π_{rj}^{d*} from origin r to node j should equal to or less than the minimal travel time π_{ri}^{d*} from origin r to node i plus the actual link travel time $\tau_a[d+\bar{\pi}_{ri}^{d}]$ in time interval $[d+\bar{\pi}_{ri}^{d}]$, where the $\bar{\pi}_{ri}^{d}$ denotes the number of time intervals traversed in π_{ri}^{d}; see definition in Section 3.2 and Equation (2). The first time interval of $[d+\bar{\pi}_{ri}^{d}]$ must be the earliest time interval that flow departing zone r in time interval d can enter link a. It follows that

$$\pi_{ri}^{d*} + \tau_a[d+\bar{\pi}_{ri}^{d*}] \geq \pi_{rj}^{d*} \qquad \forall\, a = (i,j), r, d. \tag{16}$$

If, for each O-D pair (r,s), any departure flow from origin r in time interval d enters link a at the earliest time interval $[d+\bar{\pi}_{ri}^{d*}]$, or $x_a[d+\bar{\pi}_{ri}^{d*}] \geq 0$, then the *ideal* DUO route choice conditions require that link a is on the route with minimal travel time. In other words, the minimal travel time π_{rj}^{d*} from origin r to node j should equal to the minimal travel time π_{ri}^{d*} from origin r to node i plus the actual link travel time $\tau_a[d+\bar{\pi}_{ri}^{d*}]$ in time interval $[d+\bar{\pi}_{ri}^{d*}]$. It follows that

$$\pi_{rj}^{d*} = \pi_{ri}^{d*} + \tau_a[d+\bar{\pi}_{ri}^{d*}], \quad \text{if } \; x_a^{rs*}[d+\bar{\pi}_{ri}^{d*}] > 0 \quad \forall\, a = (i,j), r, s, d. \tag{17}$$

The above equation is also equivalent to the following:

$$x_a^{rs*}[d+\bar{\pi}_{ri}^{d*}]\left[\pi_{ri}^{d*} + \tau_a[d+\bar{\pi}_{ri}^{d*}] - \pi_{rj}^{d*}\right] = 0 \quad \forall\, a = (i,j), r, s, d. \tag{18}$$

Thus, the link-time-based *ideal* DUO route choice conditions can be summarized as below:

$$\pi_{ri}^{d*} + \tau_a[d+\bar{\pi}_{ri}^{d*}] - \pi_{rj}^{d*} \geq 0 \quad \forall\, a = (i,j), r, d; \tag{19}$$

$$x_a^{rs*}[d+\bar{\pi}_{ri}^{d*}]\left[\pi_{ri}^{d*} + \tau_a[d+\bar{\pi}_{ri}^{d*}] - \pi_{rj}^{d*}\right] = 0 \quad \forall\, a = (i,j), r, s, d; \tag{20}$$

$$x_a^{rs}[d+\bar{\pi}_{ri}^{d*}] \geq 0 \quad \forall\, a = (i,j), r, s, d. \tag{21}$$

5 The Link-Time-Based VI Model

Define Ω_{ra}^{d*} as the difference between the minimal travel time from zone r to node i plus the travel time on link a and the minimal travel time from zone r to node j for flows departing from zone r in time interval d.

$$\Omega_{ra}^{d*} = \pi_{ri}^{d*} + \tau_a[d+\bar{\pi}_{ri}^{d*}] - \pi_{rj}^{d*} \tag{22}$$

The link-time-based *ideal* DUO route choice conditions are rewritten as:

$$\Omega_{ra}^{d*} \geq 0 \qquad \forall\, a = (i,j), r, d; \tag{23}$$

$$x_a^{rs*}\left[d+\bar{\pi}_{ri}^{d*}\right]\Omega_{ra}^{d*} = 0 \qquad \forall\, a = (i,j), r, s, d; \tag{24}$$

$$x_a^{rs}\left[d+\bar{\pi}_{ri}^{d*}\right] \geq 0 \qquad \forall\, a = (i,j), r, s, d. \tag{25}$$

The equivalent VI formulation of the link-time-based *ideal* DUO route choice conditions defined in Equations (23)–(25) can now be stated as follows.

$$\sum_d \sum_{rs} \sum_a \Omega_{ra}^{d*} \left\{x_a^{rs}[d+\bar{\pi}_{ri}^{d*}] - x_a^{rs*}[d+\bar{\pi}_{ri}^{d*}]\right\} \geq 0 \tag{26}$$

where $*$ denotes the DUO state, and where the dynamic traffic flow pattern must satisfy the constraints described in Equations (1)–(15).

6 Solution Algorithm

This algorithm consists a sequence of *diagonalization* iterations. First, in the initialization step, the *node time intervals* and *shortest route travel times* are initialized based on the initial link flows. Then, the algorithm solves a sequence of route choice problems (called *inner* problems) using the Frank-Wolfe (F-W) algorithm with fixed *node time intervals* until the convergence criterion of the route choice problem is satisfied. Next, the *node time intervals* and *shortest route travel times* are updated (called *outer* problems) based on the most recently assigned link flows from the *inner* problem. Adjustments of link capacities are made between the *inner* and *outer* problems to account for capacity changes caused exogenously (e.g., signal timing changes, incidents and other unexpected capacity reduction events) or generated endogenously (e.g., queue spillbacks). The algorithm terminates when the changes of the *node time intervals* obtained from two consecutive *outer* problems are within a prespecified tolerance. Figure 2 shows the steps of the solution algorithm.

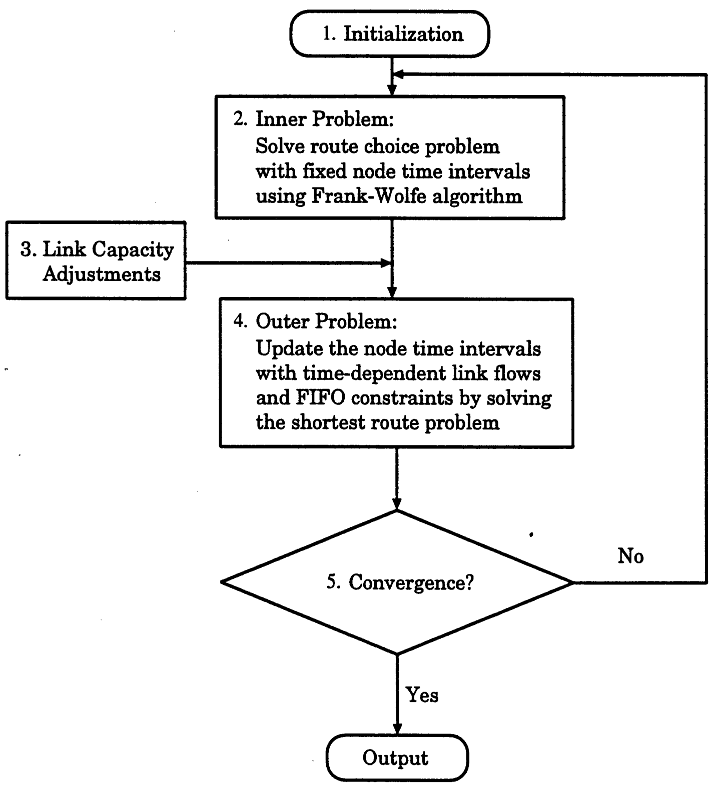

Figure 2: Flowchart of the Solution Algorithm

Steps of the solution algorithm are described as below:

Step 1: Initialization. Input the O-D matrix, time-dependent trip departure rates, network data and initial link flows (optional). With the initial link flows, the *shortest* *route travel times* and *node time intervals* are initialized as well.

Step 2: Solve Route Choice Problem (Inner Problem). Using the F-W algorithm, the route choice problem is solved to convergence with the optimal values of the *node time intervals* from the *outer* problem.

Step 3: Link Capacity Adjustments. Perform exogenous and endogenous link capacity adjustments.

Step 4: Update the Node Time Intervals (Outer Problem). Update the *node time intervals* with time-dependent link flows and FIFO constraints by solving the shortest route problem.

Step 5: Convergence Test for Outer Iterations. Sum the total number of differences of the *node time intervals* (NDIFFS) between consecutive *outer* iterations. If NDIFFS $\leq$ allowable percentage of all *node time intervals*, then the algorithm stops; otherwise, return to Step 2.

To update the *node time intervals* ($\alpha_{ri}^{d}[t]$), the following procedure is applied in Step 4:

1. find the shortest route travel times (π_{ri}^{d}) and time intervals ($\bar{\pi}_{ri}^{d}$);
2. reset values of ($\alpha_{ri}^{d}[t]$) as follows:

$$\begin{aligned}&\text{if } (\bar{\pi}_{ri}^{d} \leq t\Delta t) \text{ and } (\bar{\pi}_{ri}^{d} \geq (t-1)\Delta t)\ , \text{ then } \alpha_{ri}^{d}[t] = 1;\\ &\text{otherwise } \alpha_{ri}^{d}[t] = 0\ .\\ &\text{perform for all } r \in \mathcal{Z},\ i \in \mathcal{N};\ d = 1, \cdots \mathcal{T} \text{ and } t = d,\ d+1, \cdots \mathcal{T}\\ &\text{note: } \sum_{t=d}^{\mathcal{T}} \alpha_{ri}^{d}[t] = 1\ ;\end{aligned}$$

3. set $\pi_{rr}^{d} = d\Delta t,\ \forall r \in \mathcal{Z},\ d \in \mathcal{T}$;
4. enforce the FIFO conditions stated in Equations (9)–(12).

Note π_{rr}^{d} equals the start time of the end of the platoon departing zone r in time interval d. π_{rr}^{d} is set to $d\Delta t$ in order to set the clock correctly to the end of each time interval. Although $\alpha_{ri}^{d}[t]$ can never equal 1 when $t = 1$, flows departing in interval 1 are uniformly distributed over the previous time span ($\bar{\pi}_{ri}^{1} - \Delta t$) at each node of the network such that some flows are still assigned in time interval 1.

7 Test Network and Link Travel Time Functions

The ADVANCE Network is selected for testing the proposed DUO route choice model. Depicted in Figure 3, the ADVANCE Network is located in the northwestern suburbs of Chicago and covers about 800 square kilometers (300 square miles).

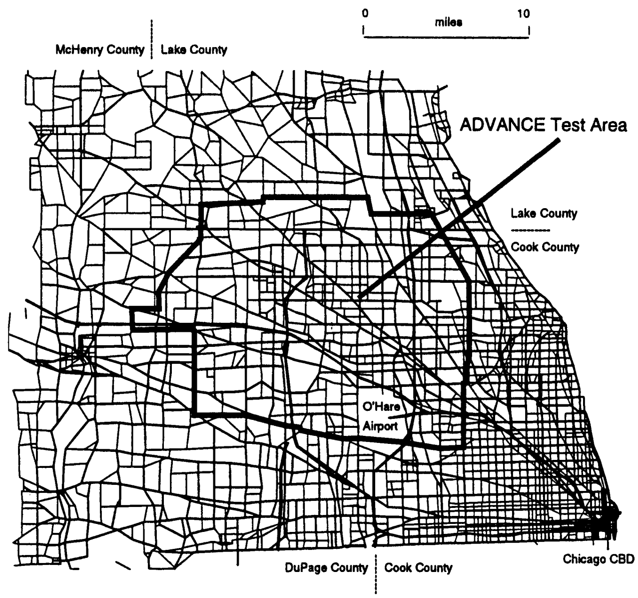

Figure 3: The ADVANCE Test Area in the Northwestern Suburbs of Chicago

Using the conventional network representation, 7,850 links and 2,552 nodes are included in the ADVANCE Network. To provide dynamic and specific link travel time estimates for the network including turning movements. a more appropriate network representation known as an *expanded intersection representation* is adopted. The *expanded intersection representation* consists of defining a special network representation so each turning movement is represented by a separate link called an *intersection link.* For example, a typical four-leg intersection with two-way approaches without any turning restrictions (U-turn excluded), four approach nodes, four exit nodes and twelve intersection links are required in this expanded network representation. Using the *expanded intersection representation*, nearly 23,000 links and 10,000 nodes are actually modeled in the solution procedure.

The specific delay function for links at signalized intersections applied in the research has the following form (Akcelik, 1988):

$$d = \frac{0.5C(1-u)^2}{1-ux} + 900T\gamma\left[x - 1 + \sqrt{(x-1)^2 + \frac{8(x-0.5)}{cT}}\right] \tag{27}$$

where d is the average delay per vehicle (second/vehicle), C is the signal cycle length (second), $u = g/C$ is the green split, g is the green time (second), $x = v/c$ is the flow-to-capacity ratio, T is the duration of the flow (hour) and $\gamma = 1$ for $x > 0.5$, and 0, otherwise. The first term, called the *uniform delay*, reflects the average delay experienced by drivers in *undersaturation conditions*, that is when the arrival flow does not exceed capacity. In oversaturation conditions, $x = 1$ is used in the uniform delay term. The second term of Equation (27), called the *overflow delay*, reflects the delay experienced by the vehicles when the flow rate is close to or exceeds the capacity.

The BPR function is temporarily used for estimating delays at major/minor priority intersections with the following form (Bureau of Public Roads, 1964):

$$t = t_0 \left[1 + 0.15 \left(\frac{v}{c} \right)^4 \right] \tag{28}$$

where t is the link travel time, v is the link flow and c is the capacity of the link at a specified level of service. Although using the BPR function as an intersection delay function is somewhat outside its designated scope, the lack of alternative functions with the desired analytical properties for use in a network equilibrium model mandates the use of the BPR function.

As for the delay function for all-way-stop intersections, the following exponential delay model is used (Meneguzzer et al., 1990).

$$d = \exp\left[3.802(v/c)\right] \tag{29}$$

where d is the average approach delay (second/vehicle), v is the total approach flow and c is the approach capacity. Equation (29) is suitable for use in a network equilibrium model, since it is defined for any flow-to-capacity ratio.

8 Computational Results and Analysis

The model is implemented on the CONVEX–C3880 at the National Center for Supercomputing Applications (NCSA), University of Illinois at Urbana-Champaign. For the ADVANCE Network with afternoon peak of two hours (4 PM to 6 PM) divided into 24 five-minute time intervals and 406,560 total travel flows are solved from a zero flow initial solution in this paper.

As shown in Figure 4, the rate of changes of the *node time intervals* is calculated as the changes in the *node time intervals* divided by the total possible changes of the *node time intervals* (i.e., *nodes* × *zones* × *intervals*; 104,061,600 total for the afternoon peak period) between consecutive *outer* iterations. According to Figure 4, it reveals that this model was solved quite smoothly for the applied travel demand over the ADVANCE Network. Note that we omit the value of the first *outer* iteration in Figure 4 to provide a better display of the variations in the rest of iterations.

Flow profile of the selected approach link and the time-dependent flow departure rates are displayed in Figures 5. In this research, time-of-day (e.g., afternoon peak) O-D tables were factored from CATS (Chicago Area Transportation Study) estimates for 1990. To solve this model, the afternoon peak period is further divided into 24 five-minute time intervals. Besides, the five-minute flow departure rates for each origin zone (447 zones total) are derived from half-hour departure rates provided by CATS. Therefore, the time-dependent travel demand is loaded onto the network by multiplying the time-dependent flow departure rates (sum to one) and the afternoon peak O-D table. The highest flow departure rates appear between the eleventh and twelfth intervals (4:50 to 5:00 PM). Based on the derived flow departure rates, flows on the selected link are obtained and plotted. In the first few time intervals, the link flows tend to be low because the solution algorithm started from an empty

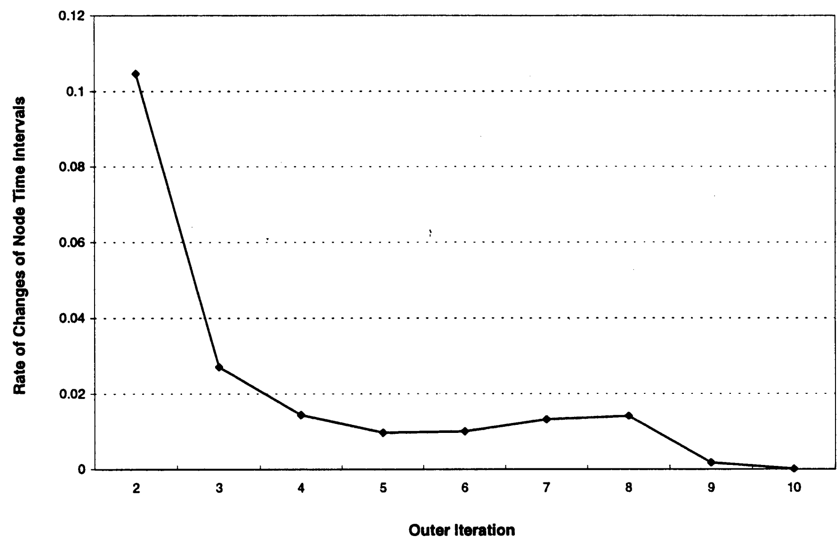

Figure 4: Rate of Change of Node Time Intervals of the Afternoon Peak Period

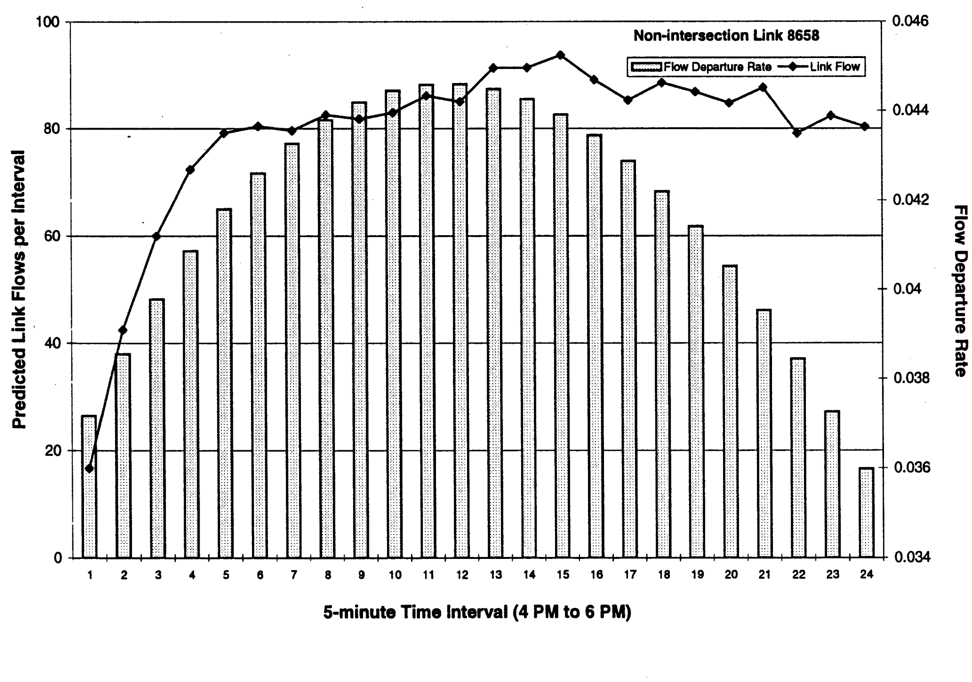

Figure 5: Predicted Link Flows vs. Flow Departure Rates per Time Interval

network. The oscillation of flows in the last few time intervals implies the occurrence and dissipation of the queue spillback of this observed link. Unfortunately, empirical link flow and flow departure rate data are not available for the ADVANCE Network, either in general, or more specifically for the O-D matrix used in this solution. These data, as well as route flow data, are urgently needed to advance the state of the art of network modeling for ITS.

9 Conclusions

This paper presents a discrete-time formulation of the link-time-based variational inequality model of dynamic user-optimal route choice for evaluating time-dependent traffic characteristics for Advanced Traffic Management Systems (ATMS) and Advanced Traveler Information Systems (ATIS) such as the recently concluded ADVANCE Project.

To date, very few traffic flow prediction models are suitable for ATMS and ATIS applications. Although not yet fully validated, this model is able to predict time-dependent traffic characteristics for a large-scale traffic network which are reasonable and internally consistent. Eventually, dynamic route choice models should be integrated into a traffic management center to support the decisions on the adjustments of arterial signal timing, ramp metering, incident management and future route guidance strategies, etc.

This paper describes the largest dynamic route choice model that has been solved to date, to the best knowledge of the authors. Consequently, this research establishes a new benchmark for dynamic route choice modeling. Solving dynamic route choice models on a realistic and large-scale traffic network is no longer an intractable task.

Acknowledgments

The authors are pleased to acknowledge the following sources of support in the preparation of this chapter: the Illinois Department of Transportation and the Federal Highway Administration through the ADVANCE Project; the Institute of Transportation of the Ministry of Communications, Taiwan, Republic of China, in support of the ADVANCE Project; the National Institute of Statistical Sciences; the National Center for Supercomputing Applications, University of Illinois at Urbana-Champaign, for the use of computational facilities. The last two organizations are supported by the National Science Foundation, Washington, D.C.

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